15 th Marcel Grossmann
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# Physical Effects of Massless Cosmic Strings in Expanding Universe

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# Plan

- massless strings (MCS) in a flat space: the beauty of the special relativity;

- the technique, wake and Kaiser-Stebbins effects for MCS;

- field theory;

- MCS in expanding universe (focus on de Sitter);

- effects, observations

# Basic facts:

- worldsheet of a cosmic string is an extremal surface in a given background geometry (Nambu-Goto eqs);

$$X^\mu = X^\mu(\sigma,\tau)$$

extrinsic curvatures vanish

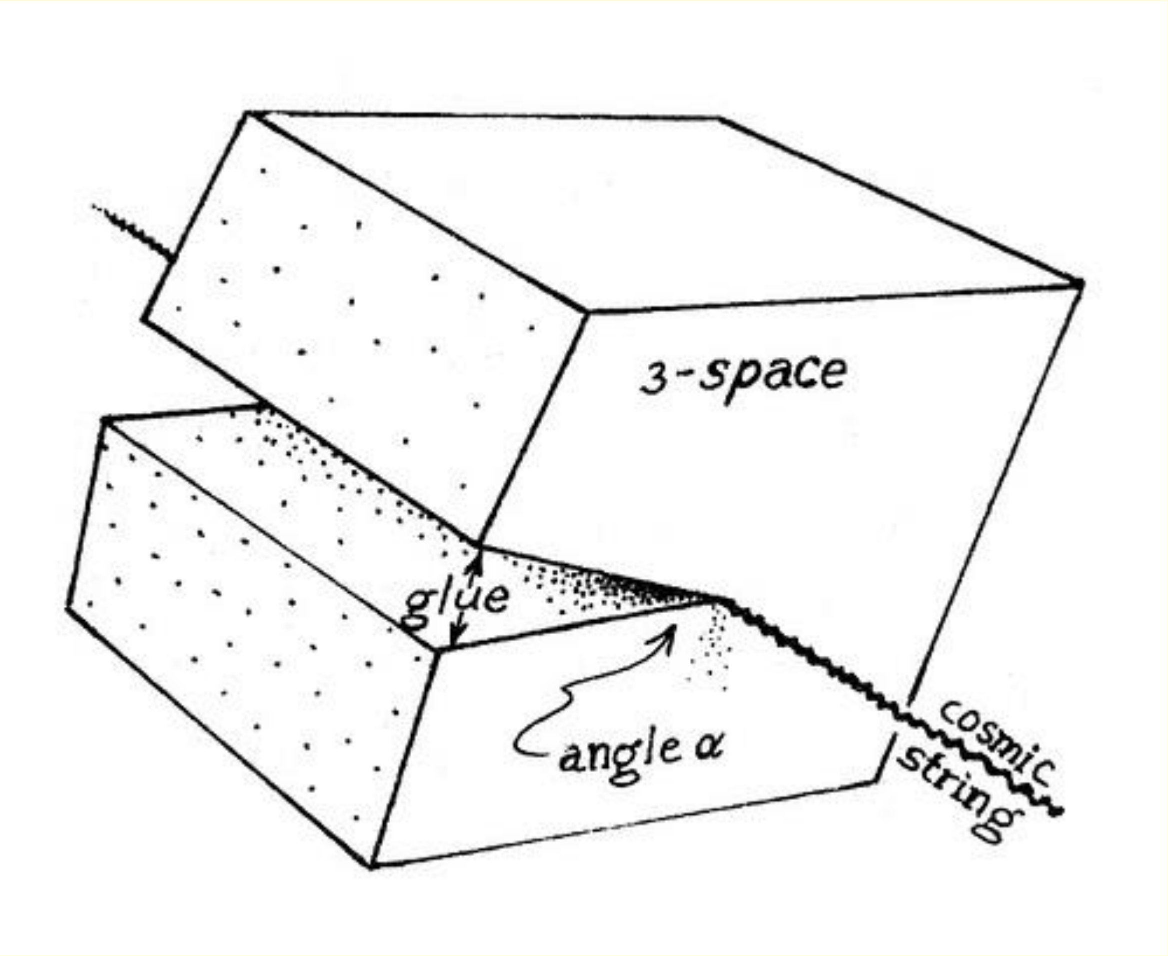
- gravitational effects of the string are global (no local changes in the geometry)

$$ds^2 = -dt^2 + dz^2 + \alpha^2 r^2 d\varphi^2 + dr^2 \quad ,$$

$$0 \le \varphi < 2\pi$$

$$\alpha = 1 - 4\mu G, \quad 0 < \alpha < 1$$

$\mu$ is the string tension, when the string is at rest



$$ds^2 = -dt^2 + dz^2 + r^2 d\bar{\varphi}^2 + dr^2 \quad ,$$

$$0 \le \bar{\varphi} < 2\pi\alpha$$

# Moving string and the Penrose limit (Aichelburg-Sexl boost)

use 'continuous' coodinates

$$ds^2 = -dt^2 + dz^2 + dx^2 + dy^2 + (\alpha^2 - 1)\frac{(xdy - ydx)^2}{x^2 + y^2}$$

go to the frame where the string moves with the velocity $v$ along the $x$-direction

$$x = \gamma(x' - vt') \ , \ \gamma = \frac{1}{\sqrt{1 - v^2}} \qquad (c = 1)$$

the Penrose limit (an 'ultra boost'): $v$ approaches the velocity of light, $\mu \to 0$ (massless string) with a finite energy (per length)

$$\boxed{E \equiv \lim_{v \to 1} (\gamma\mu)}$$

# Metric in the Penrose limit

$$ds^2 = -dt^2 + dz^2 + dx^2 + dy^2 + (\alpha^2 - 1)\frac{(xdy - ydx)^2}{x^2 + y^2}$$

$$\gamma(\alpha^2 - 1) \to -8GE$$

$$-dt^2 + dx^2 = -dudv \quad , \quad u = t' - x' \quad , \quad v = t' + x'$$

$$\gamma^{-1}x \to -u \quad ,$$

$$\frac{\gamma}{x^2 + y^2} \to \frac{\pi}{|y|}\delta(u)$$

$$ds^2 = -dudv + dz^2 + dy^2 - 8\pi GE|y|\delta(u)du^2$$

Israel, Barrabes, Hogan (2002), van de Meent (2012)

# Stress-energy tensor of a massless string

$$T_{\mu\nu} = \rho(U_\mu U_\nu - l_\mu l_\nu)$$

$$\rho = \mu\delta(y)\delta(x)$$

the Penrose limit:

$$x \to \gamma u \quad , \quad \gamma\delta(x) \to \delta(u) \quad ,$$

$$\gamma\mu \to E \quad , \quad \gamma^2 \rho = \tilde{\rho} \quad ,$$

$$\gamma^{-1} U_\mu \to u_\mu \quad , \quad u^2 = 0$$

$$\boxed{\begin{aligned} T_{\mu\nu} &= \tilde{\rho}\, u_\mu u_\nu \quad , \\ \tilde{\rho} &= E\delta(y)\delta(u) \end{aligned}}$$

$u_\mu$ is the velocity of the string

# Do the metric and the stress-energy correspond each other?

$$ds^2 = -du\,dv + dz^2 + dy^2 - 8\pi G E |y| \delta(u) du^2$$

regularization

$\delta(u) \to \chi(u)$ a smooth function

$$\boxed{ds^2 = -du\,dv + dz^2 + dy^2 - 8\pi G E |y| \chi(u) du^2}$$

exact solution to the problem

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu} = \tilde{\rho}\, u_\mu u_\nu \quad , \quad \tilde{\rho} = E\delta(y)\chi(u)$$

$\chi(u)$ is a 'profile' of a family of strings moving one after another

# Deviation of geodesics by the moving string

Let $O(\Delta\varphi)$ be a mutual rotation of parallel geodesics after passing the string from different sides (in the frame, where the string is at rest)

Let $L(v)$ be the Lorentz boost to the frame where the string is moving with the velocity $v$.

The mutual rotation of the parallel geodesics by the moving string is

$$O'(\Delta\varphi) = L(v)O(\Delta\varphi)L^{-1}(v)$$

The boost is

$$v \to 1, \quad \gamma\Delta\varphi \to 8\pi GE \equiv \lambda$$

The mutual rotation of the parallel geodesics by the massless string is

$$\lim_{v\to 1} O'(\Delta\varphi) = M(\lambda)$$

# Transformations by the string gravity

Let $O(\Delta\varphi)$ be a mutual rotation of parallel geodesics after passing the string

Use the following representation for rotation of trajectories with $y < 0$

$$O(\alpha) = I + (\cos\alpha - 1)\left(p_x \times \tilde{p}_x + p_y \times \tilde{p}_y\right) +$$

$$\sin\alpha\left(p_x \times \tilde{p}_y - p_y \times \tilde{p}_x\right)$$

$\alpha = 8\pi G\mu, \quad \tilde{p}$ - vector with lower indices

The boosted matrix

$$O'(\alpha) = I + (\cos\alpha - 1)\left(p'_x \times \tilde{p}'_x + p_y \times \tilde{p}_y\right) +$$

$$\sin\alpha\left(p'_x \times \tilde{p}_y - p_y \times \tilde{p}'_x\right) \quad , \qquad \text{where} \qquad p'_x = \gamma\left(p_x + v p_0\right)$$

$$M(\lambda) = \lim_{v \to 1} O'(\alpha) = I - \frac{\lambda^2}{2} u \times \tilde{u} - \lambda p_y \times \tilde{u} + \lambda u \times \tilde{p}_y$$

$$u = p_0 + p_x$$

If a string moves between an observer and a test particle (light) with the initial velocity $\bar{U}^{\mu}$, the obsever with $y>0$ registers the particle (light) coming from the region $y<0$ with a changed velocity $U^{\mu}=M^{\mu}{}_{\nu}(\lambda)\bar{U}^{\nu}$

## Parabolic subgroup of the Lorentz group

$$
\begin{aligned}
U^{u} &= \bar{U}^{u} \\
U^{v} &= \bar{U}^{v} + 2\lambda\bar{U}^{y} + \lambda^{2}\bar{U}^{u} \\
U^{y} &= \bar{U}^{y} + \lambda\bar{U}^{u} \quad , \\
U^{z} &= \bar{U}^{z}
\end{aligned}
$$

one has an Abelian subgroup of the Lorentz group - a hyperbolic subgrop

$$
M(\lambda_1)M(\lambda_2)=M(\lambda_1+\lambda_2)
$$

# ‘Stationary’ coordinates for Parabolic subgroup

Transformations

$$u' = u \quad , \quad v' = v + 2\lambda y + \lambda^2 u$$

$$y' = y + \lambda u \quad , \quad z' = z$$

imply that

$\sigma = v - y^2 / u$ is invariant, and

$\zeta' = \zeta + \lambda ,$ where $\zeta = y / u$

in new set of coordinates $(u, \sigma, \zeta, z)$ metric changes to

$$\boxed{ds^2 = -du d\sigma + u^2 d\zeta^2}$$

Killing vector field $\partial_\zeta$

# straightforward derivation of geodesic deviations

We use the metric :

$$ds^2 = -du dv + dz^2 + dy^2 - \lambda |y| \chi(u) du^2$$

coordinates $u, v, y, z$ are global, but geodesics are not straight lines in these coordinates, integration yields for the components of the 4-velocity ($y \neq 0$, $u \neq 0$,):

$$u^u = \bar{u}^u, \quad u^z = \bar{u}^z,$$

$$u^y = \bar{u}^y - \frac{\lambda}{2} \varepsilon(y) \int_{-\infty}^{u} du' \chi(u') \, \bar{u}^u,$$

$$u^v = \bar{u}^v - \lambda |y| \chi(u) \, \bar{u}^u - \lambda \varepsilon(y) \int_{-\infty}^{u} du' \chi(u') \, \bar{u}^y +$$

$$+ \frac{\lambda^2}{2} \int_{-\infty}^{u} du' \chi(u') \int_{-\infty}^{u} du'' \chi(u'') \, \bar{u}^u;$$

where $\bar{u}^u, \bar{u}^v, \bar{u}^z, \bar{u}^y$ are initial values of the 4-velocity

# string ‘horizon’

In the limit $\chi(u) \to \delta(u)$ :

$$u^{\mu} = M^{\mu}{}_{\nu}\left(-\varepsilon(y)\theta(y)\lambda/2\right)\bar{u}^{\nu}$$

this is a 'rotation' w.r.t. coordinate chart,

mutual rotation w.r.t. initial values:

$$\bar{u}^{\mu}_{+} = M^{\mu}{}_{\nu}\left(\theta(y)\lambda\right)\bar{u}^{\mu}_{-}$$

there is no 'rotation' till objects cross the null surface $u = 0$

this null surface has a meaning of the string horizon: string cannot affect an objects under the horizon $u < 0,$ since the future light cone of any point of the string lies above the horizon (and it is tangent to it)

# The rules in CS spacetimes:

'left' trajectories : start from CS horizon at $y < 0$

'right' trajectories : start from CS horizon at $y > 0$

(both types of trajectories are transformed w.r.t. the coordinate chart after crossing the horizon)

on a coordinate chart where 'right' trajectories are not changed after CS horizon

'left' trajectories look as starting from the horizon with :

- transformed coordinates
- transformed 4 - velocities

# Properties of parabolic transformations

1. $M^{\mu}{}_{\nu}(\lambda)u^{\nu} = u^{\mu}$, where $u^{\mu}$ is the 4-velocity of the string,

the string world-sheet is left invariant; **'null rotations'**

2. $M(\lambda)$ determines transformation of components of a vector under

a parallel transport around the string ($M(\lambda)$ is a holonomy);

3. transformation of coordinates:

$$\bar{x} = \left(1 - \frac{\lambda^2}{2}\right)x + \frac{\lambda^2}{2}t + \lambda y$$

$$\bar{t} = -\frac{\lambda^2}{2}x + \left(1 + \frac{\lambda^2}{2}\right)t + \lambda y$$

$$\bar{y} = y + \lambda(t - x) \quad , \quad \bar{z} = z$$

# Physical effects: spatial mutual movements (analogue of the wake effect)

1. mutual movement of coordinate systems on different sides of the string:

$\bar{x}^{\mu}$ moves w.r.t. $x^{\mu}$ with the coordinate velocity

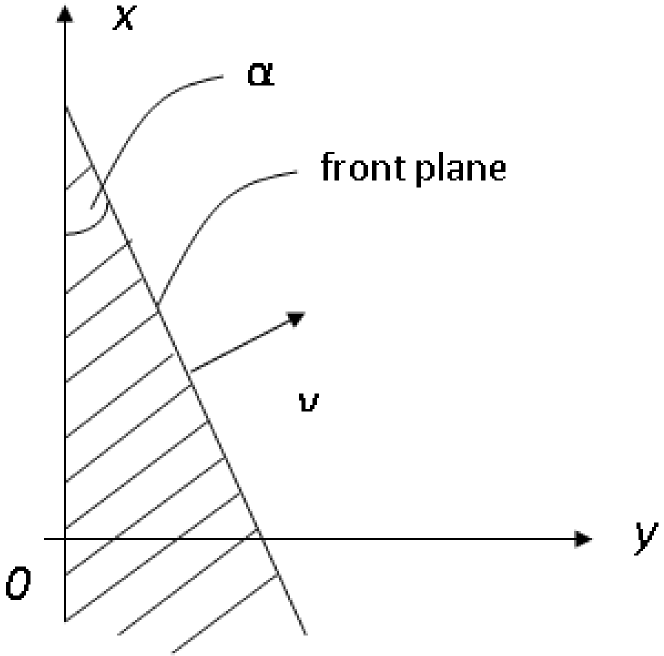
$$v_x = \frac{\lambda^2}{2+\lambda^2}, \quad v_y = \frac{2\lambda}{2+\lambda^2}, \quad |v| = \lambda \frac{\sqrt{4+\lambda^2}}{2+\lambda^2} \simeq 8\pi GE$$

- two observers which are at rest w.r.t. each other start to move toward each other,

if a massless string passes between them;

- images of a star (in the lensing effect) move toward each other;

# Physical effects: spatial mutual movements (analogue of the wake effect)



$$v^y = \frac{\bar{v}^y + \lambda(1-\bar{v}^x)}{\left(1+\frac{\lambda^2}{2}\right) - \frac{\lambda^2}{2}\bar{v}^x + \lambda\bar{v}^y}$$

$$\delta v^y \simeq 8\pi GE \quad (GE \ll 1)$$

velocity of the wake front is $v = \frac{1}{\sqrt{1+\lambda^2}}$

# Physical effects: shifts of spectra
# (analogue of the Kaiser-Stebbins effect)

2. gamma quantum 4-momentum:

$$\bar{k}^{\mu} = M^{\mu}_{\ \nu} k^{\nu} \quad , \qquad k^2 = \bar{k}^2 = 0$$

$$\bar{\omega} = \bar{k}^{t} = \left(1 + \frac{\lambda^2}{2}\right)\omega - \frac{\lambda^2}{2} k^{x} + \lambda k^{y}$$

- $\bar{\omega} = \omega$ , if the quantum moves strictly in the same direction as the string,
- $\bar{\omega} = \left(1 + \lambda^2\right)\omega$ , if the quantum moves strictly in the opposite direction,
- $\bar{\omega} = \left(1 + \lambda + \frac{\lambda^2}{2}\right)\omega$ , if the quantum moves perpendicularly to the

direction of the string motion

# Physical effects: shifts of gamma spectra (cond)

General case of the energy of gamma quantum:

$$\bar{\omega} = \bar{k}^t = \left(1 + \frac{\lambda^2}{2}\right)\omega - \frac{\lambda^2}{2}k^x + \lambda k^y = \omega f(\lambda)$$

$$f(\lambda) = 1 + \frac{\lambda^2}{2}(1 - n^x) + \lambda n^y,$$

$$n^x = r\cos\varphi,\ \ n^y = r\sin\varphi \quad , \quad 0 < r \le 1$$

introduce $\cos\alpha = \dfrac{\lambda/2}{\sqrt{1+\lambda^2/4}},$ then

$$f(\lambda) = \lambda r\sqrt{1+\lambda^2/4}\left(\frac{1+\lambda^2/2}{\lambda r\sqrt{1+\lambda^2/4}} - \cos(\varphi+\alpha)\right) > 0$$

since $\dfrac{1+\lambda^2/2}{\lambda r\sqrt{1+\lambda^2/4}} > 1$

# More on optical effects

# Setup

We use Minkowsky coordinates $t, x, y, z$

String trajectory (worldsheet) $y = 0, \quad u = t - x = 0$

String horizon $u = 0$

Position of observer $x_0 = 0, \quad y_0 = a > 0$

Equation for the past light cone of an observer at a point $\vec{x}_o$ at a moment $t_o$

$$\vec{x}(t) = \vec{x}_o + (t - t_o)\vec{n}\ , \quad \vec{n}^2 = 1\ , \quad t < t_o$$

we put $n^x = \sin\theta\cos\varphi, \ n^y = \sin\theta\sin\varphi, n^z = \cos\theta$ ,

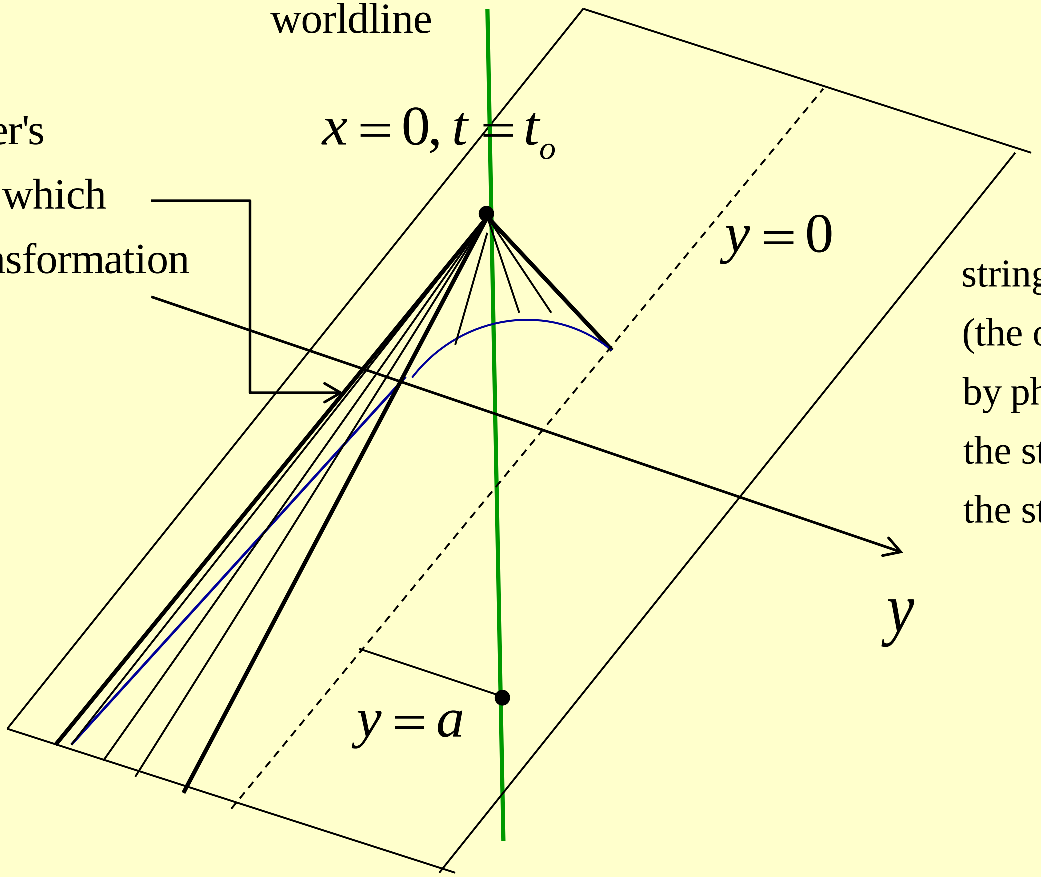
intersection of the light cone with a string is $x(t) = t$ , $y(t) = 0$

Directions $\vec{n}$ of make an image of the string seen by the observer.

$\vec{n}$ lies on a cone (a string visibility cone). All quanta transformed by the string are inside the visibility cone.

# Image of a string seen by observer

For an observer, string gravity changes direction and energy of quanta coming from the other side of the string:



observer's
worldline

part of observer's
past lightcone which
undergoes transformation

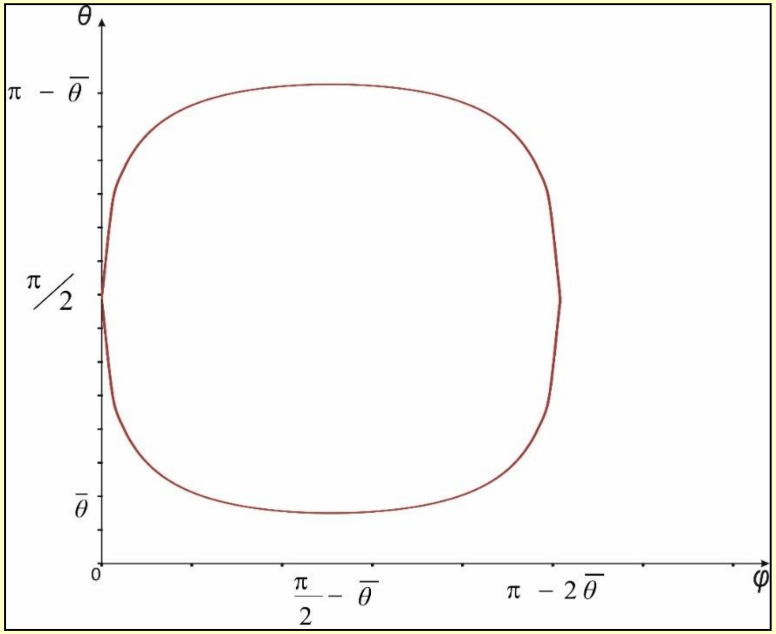
string 'horizon' $u = 0$
(the observer cannot be affected
by physical processes caused by
the string utill it crosses
the string 'horizon')

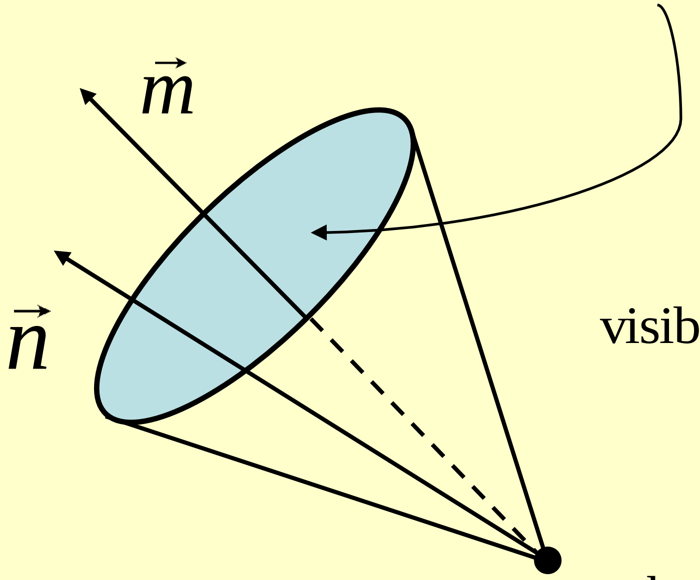
# Image of a string

equation for the "image" of the string:

$$\sin\theta\sin(\varphi+\bar{\theta})=\sin\bar{\theta}\equiv\frac{a}{\sqrt{a^2+t_o^2}},\quad (\vec{n}\cdot\vec{m})=\cos\hat{\theta},\quad \hat{\theta}\equiv\frac{\pi}{2}-\bar{\theta}$$

axis of the visibility cone $\vec{m}$ is $m_x=\cos\hat{\theta},\ m_y=\sin\hat{\theta}$





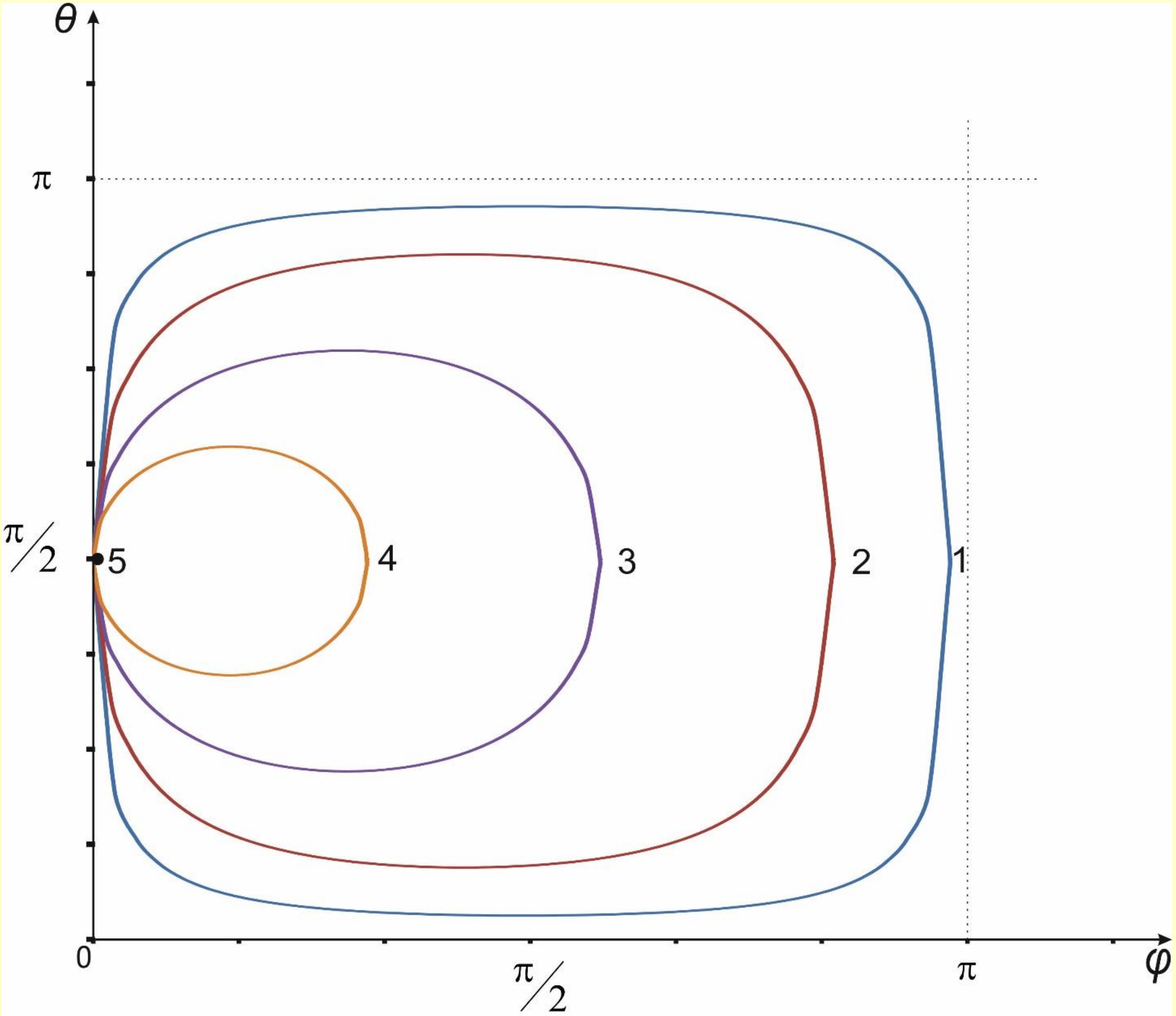
transformed quanta are inside

visibility cone

observer

# Image of a string (continued)

time evolution of the "image" (for an observer at rest)

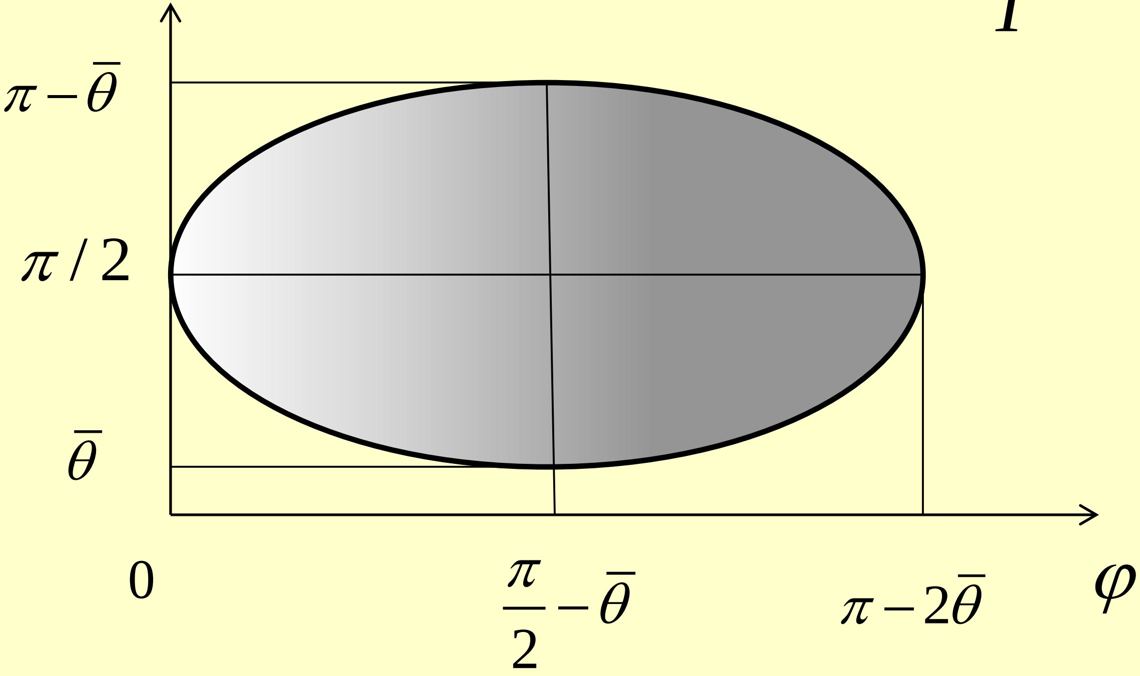


# Anisotropy formula

$$\boxed{\frac{\delta T}{T}=\frac{\omega_o-\omega_e}{\omega_e}=\frac{\lambda^2(n_x-1)+2\lambda n_y}{2-\lambda^2(n_x-1)-2\lambda n_y}\Theta(\vec{n}),}$$

$$\Theta(\vec{n})=\begin{cases}1, & \vec{n}\ \text{ is inside the string image}\\ 0, & \vec{n}\ \text{ is outside the string image}\end{cases}$$

at small string energies $\lambda \ll 1\,(E \ll 1/G)$ $\quad \dfrac{\delta T}{T}=\lambda n_y\Theta(\vec{n})$



# Field theory on MCS spacetime

# General rules in MCS spacetimes:

'left' observers cross MCS horizon at $y < 0$

'right' observers cross MCS horizon $y > 0$

$R$ coordinate charte : left trajectories transform, right don't

$L$ coordinate charte : right trajectories transform, left don't

No discontinuities in physical data on the horizon for the observers :

this implies that all physical quantities transform by the holonomy

at the horizon at $y > 0$ in $L$ coordinate, or at $y < 0$ in $R$ coordinate

# Horizon data (in R-coordinates):

$\varphi_{in}(x)_{u\to 0^-}$ - 'incoming' data on the horizon

$\varphi_{out}(x)_{u\to 0^+}$ - 'outcoming' data on the horizon

horizon acts as a Cauchy surface,

in $R$ coordinate charte (as an example) :

$$\varphi_{out}(x)_{u\to 0^+} = \varphi_{in}(x)_{u\to 0^-} \quad , \; y > 0$$

$$\varphi_{out}(x)_{u\to 0^+} = \bar{\varphi}_{in}(x)_{u\to 0^-} \quad , \; y < 0$$

$\bar{\varphi}_{in}$ is a holonomy transform of $\varphi_{in}$

# Transformation of plane waves:

$$\varphi_{in}(x) = e^{ik_\mu x^\mu} \quad , \quad k^2 = 0$$

$$\varphi^{+}_{out}(x)_{u\to 0^+} = e^{ik_\mu x^\mu}, \quad y > 0$$

$$\varphi^{-}_{out}(x)_{u\to 0^+} = e^{ik_\mu \bar{x}^\mu} = e^{i\bar{k}_\mu x^\mu}, \quad y < 0,$$

$$\bar{x}^\mu = M^\mu_\nu(-\lambda)x^\nu, \quad \bar{k}^\mu = M^\mu_\nu(\lambda)k^\nu$$

for $k_v = k_y = k_z = 0$ (wave propagating exactly in the direction of the string movement):

$$\varphi_{in}(x) = e^{ik_u u} = \varphi_{out}(x)$$

no transformation on the horizon

# Plane waves:

if $k_v \neq 0$ , calculations show that $(a = k_y/(2k_v))$:

$$\varphi_{out}(x) = \theta(y + au)e^{ik_\mu x^\mu} + \theta(-(y + au - \lambda u))e^{i\bar{k}_\mu x^\mu} + \tilde{\varphi}(x)$$

$$\tilde{\varphi}(x)_{u=0} = 0, \quad \tilde{\varphi}(x) \sim \lambda\sqrt{\frac{|k_v|}{u}}$$ , at large $u$

interference in the region $0 \leq y + au \leq \lambda u$

$$\tilde{\varphi}(x) = \frac{i}{\pi} e^{i\left(k_z^2 u/(2k_v)+k_v(v-y^2/u)+k_z z\mp\pi/4\right)} \int_0^\infty dx\, e^{-ux^2/(4k_v)} \times$$

$$\left[\frac{k_-}{x^2 - ik_-^2} - \frac{k_+}{x^2 - ik_+^2}\right]$$ , $k_+ = k_y + 2k_v y/u,\ k_+ = k_+ - 2\lambda k_v$

# Equivalence between R- and L-coordinates:

in $R$ – coordinates :

$$\varphi_{out}(x)\Big|_{u\to 0^+} = \varphi_{in}(x)\Big|_{u\to 0^-} \quad , \; y > 0$$

$$\varphi_{out}(x)\Big|_{u\to 0^+} = \varphi_{in}(M(-\lambda)x)\Big|_{u\to 0^-} \quad , \; y < 0$$

coordinate transform $x' = M(-\lambda)x$

yields horizon data in $L$ – coordinates :

$$\varphi'_{out}(x')\Big|_{u\to 0^+} = \varphi_{in}(M(\lambda)x')\Big|_{u\to 0^-} \quad , \; y > 0$$

$$\varphi'_{out}(x')\Big|_{u\to 0^+} = \varphi_{in}(x')\Big|_{u\to 0^-} \quad , \; y < 0$$

where $\varphi'_{out}(x') = \varphi_{out}(x)$

this implies covariance of the equations w.r.t. the Lorentz group

# MCS in expanding Universe

# Massive strings in de Sitter

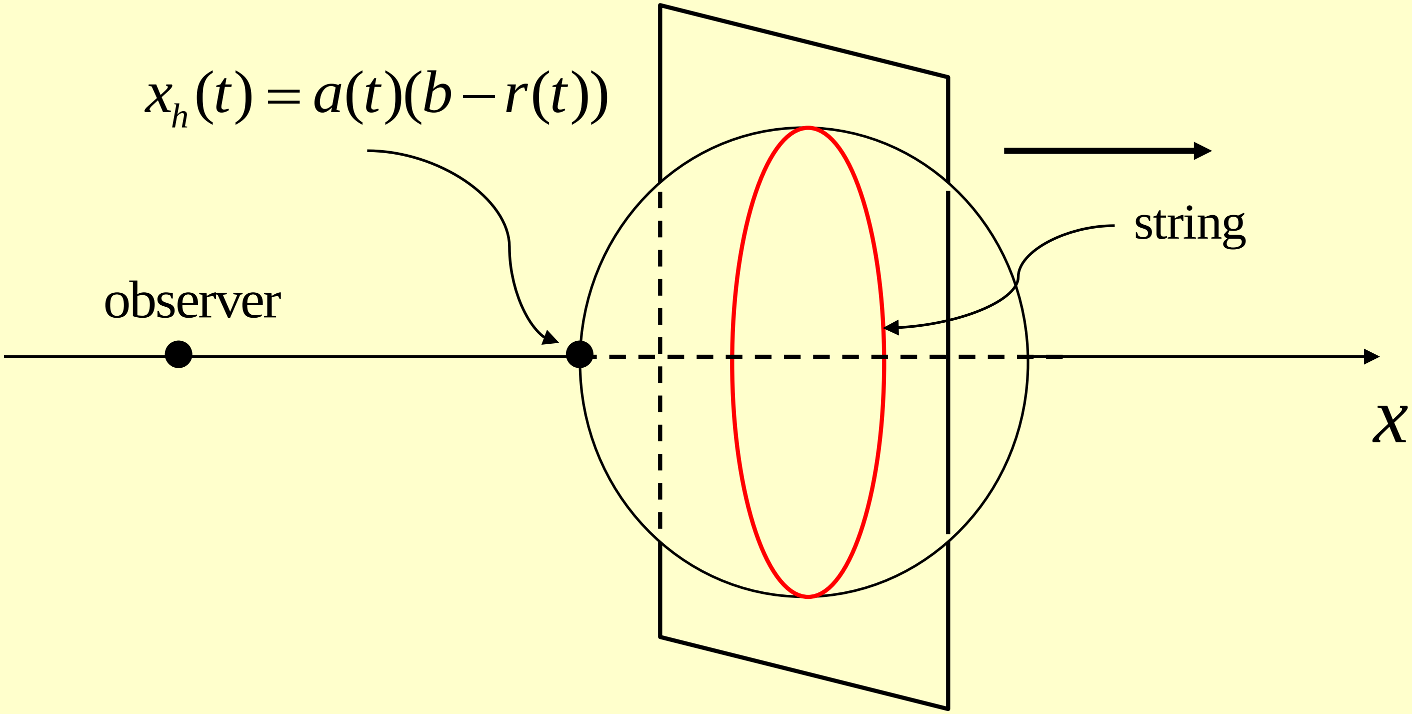
massive strings in de Sitter have been studied by: Linet (1986),

de Vega and Sanchez (1993), and later by a number of authors:

massive string is a cirle which shrinks and expands with the de Sitter universe

It makes sense to study closed massless strings in expanding Universe

# 2D null surfaces in a flat Universe



$$ds^2=-dt^2+a^2(t)(dx^2+dy^2+dz^2)$$

cosmological horizon: $(x-b)^2+y^2+z^2=r^2(t), \qquad r(t)=\int^t \frac{dt'}{a(t')}$

the string world sheet could be a 2D null surface which is intersection of the cosmological horizon and a plane $\alpha x+\beta y=0$

- a model of a circular string which moves from the left to the right
- string horizon = cosmological horizon

to realize this model one needs a global Killing vector field

which generates parabolic transformations near string world sheet

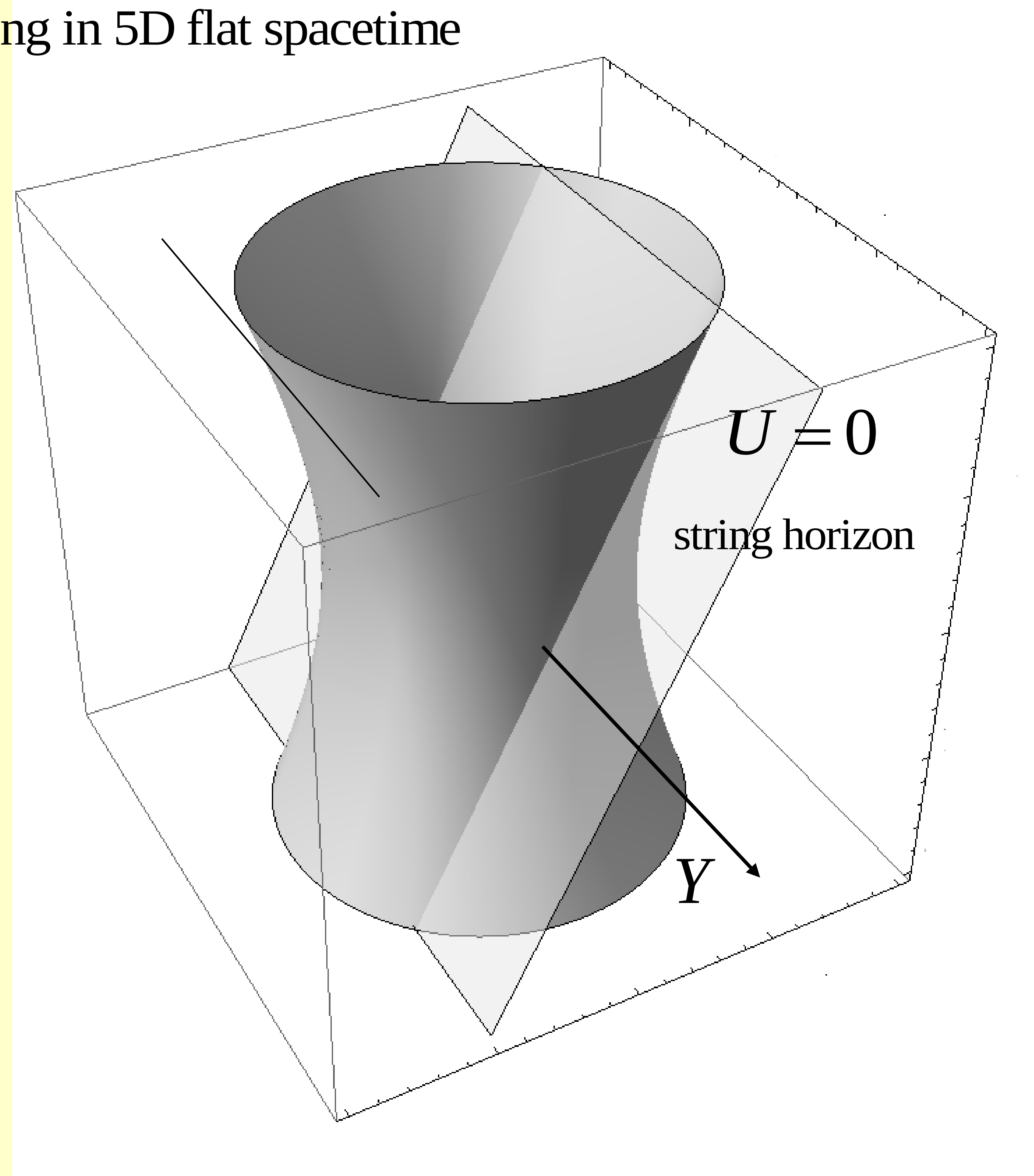
and allows one to constract nontrivial  holonomy

we demonstrate how it can be done in de Sitter universe

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

$$a(t) = e^{\chi}, \quad \chi = t / a$$

use embedding in 5D flat spacetime



## Constructing massless string in de Sitter

$\mathbb{R}^{1,4}$ : $ds_5^2 = -dUdV + dY^2 + dX_2^2 + dX_3^2$ ,

$U = X^0 - X^1, \quad V = X^0 + X^1,$

embedding: $-UV + Y^2 + X_2^2 + X_3^2 = a^2$

string (brane) horizon $U = 0,$ string (brane) position $Y = 0$

string worldsheet in de Sitter: $S^1 \times R^1$

use 5-dimensional parabolic subgroup $M_5(\lambda)$ of $O(1,4)$

which leaves $U = Y = 0$ invariant

# Holonomy

- rotation of a vector tangent to de Sitter under a parallel transport around a string worldsheet $U = Y = 0$ is determined by $M_5(\lambda)$;

- $M_5(\lambda)$ at $U = Y = 0$ maps the tangent space to a tangent space (normal vector at the string wourldsheet does not change under $M_5(\lambda)$;

- $M_5(\lambda)$ acts as $M(\lambda)$ on the tangent space at $U = Y = 0$

# MCS in a flat de Sitter  universe

$$X^0 = a\sinh\chi + \frac{r^2}{2a}e^{\chi}, \quad X^1 = xe^{\chi}, \quad X^3 = ze^{\chi}, \chi \equiv \frac{t}{a}$$

$$X^2 = \sin\alpha\left(a\cosh\chi - \frac{r^2}{2a}e^{\chi}\right) + \cos\alpha\ ye^{\chi}$$

$$Y = \cos\alpha\left(a\cosh\chi - \frac{r^2}{2a}e^{\chi}\right) - \sin\alpha\ ye^{\chi}$$

this yields metric: $ds^2 = -dt^2 + e^{2\chi}(dx^2 + dy^2 + dz^2)$

observer at a  centre of coordinates $X^0(t) = a\sinh\chi, \ X^p(t) = a\cosh\chi\, M^p,$

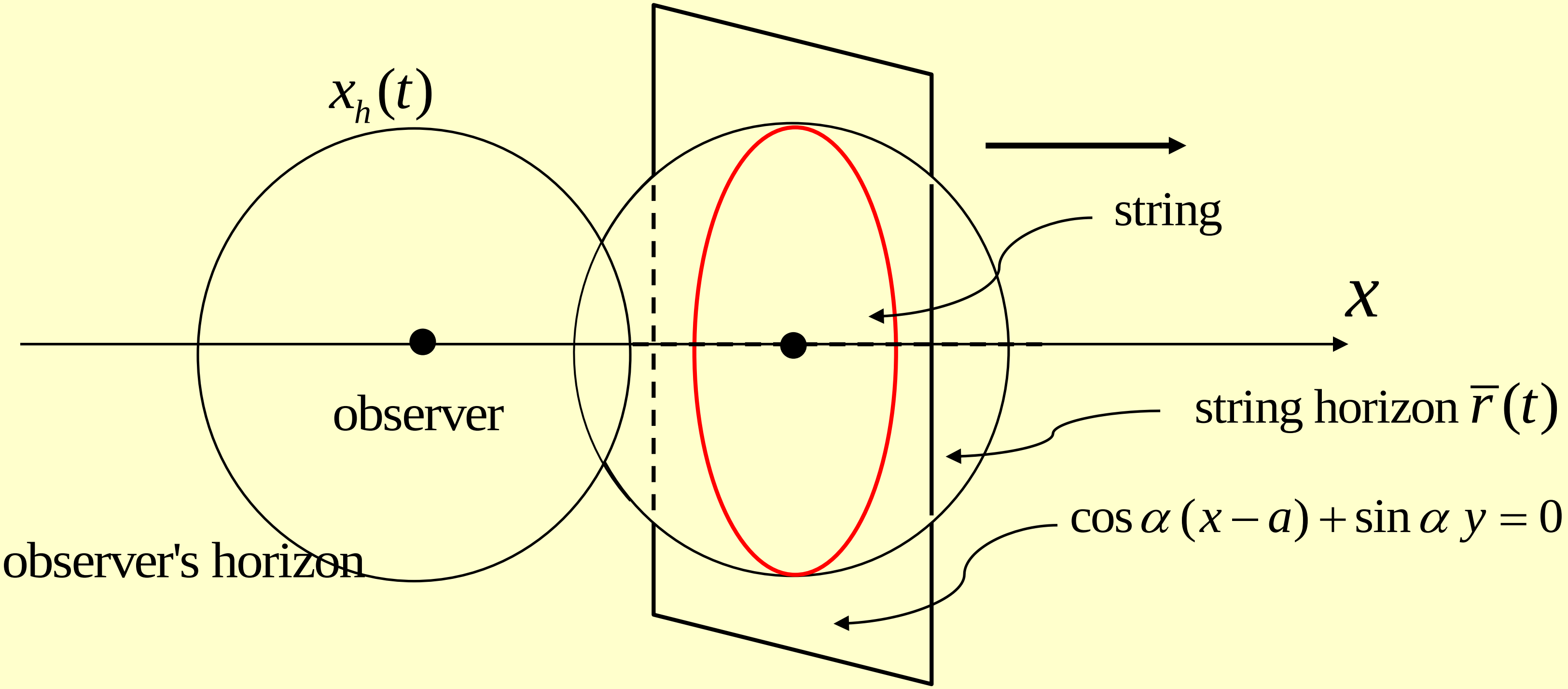
with $M^Y = \cos\alpha, M^2 = \sin\alpha$

string worldsheet:

$$U = 0\ :\quad (x-a)^2 + y^2 + z^2 = \bar{r}^2(t), \quad \bar{r}(t) = e^{-\chi}a$$

$$Y = 0 \bigcap U = 0\ :\quad \cos\alpha\,(x-a) + \sin\alpha\ y = 0$$

# MCS in a flat de Sitter Universe



- string (cosmological) horizon: $(x-a)^2+y^2+z^2=\bar{r}^2(t), \quad \bar{r}(t)=ae^{-\chi}$
- physical distnance between observer and string horizon $x_h(t)=a\,e^{\chi}-a$
- observer crosses horizon at $t=0$
- $\frac{\pi}{2}-\alpha$ is an angle between velocity of the string and the plane of the string
- 'left' side of the horizon: $\cos\alpha\,(x-a)+\sin\alpha\;y>0$

# Parabolic subgroup of O(1,4)

parabolic transformations:

$$U' = U, \quad V' = V + \lambda^2 U + 2\lambda Y, \quad Y' = Y + \lambda U,$$

$$X_2' = X_2, \quad X_3' = X_3,$$

5 D Killing vector field : $\delta X^L = \lambda \zeta^L$

induced Killing field on de Sitter: $\zeta^\mu = g^{\mu\nu} X^L{}_{,\nu} \zeta_L$

Killing field on string horizon: $\bar{\zeta}^\mu = -g^{\mu\nu} U_{,\nu} Y \big|_{U=0}$

For 'left' $(Y<0)$ matter crossing the string horizon holonomy transformations are: $\delta x^\mu = \lambda \bar{\zeta}^\mu \quad , \quad \delta u^\mu = \lambda L_\zeta u^\mu$

# Killing vector field

Computations yield:

$$\zeta^0 = -(n\cdot\bar{x}), \quad \zeta^i = \frac{x^i}{a}(p\cdot\bar{x}) - n^i e^{-\chi} U$$

$$p^i \equiv (\cos\alpha, \sin\alpha, 0)$$ - normal vector to the string plane $(p\cdot\bar{x}) = 0$

$$\bar{x}^i = (x-a, y, z),$$

$$U = \frac{e^\chi}{2a}\left(\bar{x}^i\bar{x}^i - a^2 e^{-2\chi}\right)$$

one can check that:

$$\zeta^{\mu;\nu} + \zeta^{\nu;\mu} = 0\,,$$

$$\zeta^\mu = 0 \quad \text{, if } (p\cdot\bar{x}) = U = 0$$

# Physical effects

# Wake effects (small energies of MCS)

'left' $(Y<0)$ freely moving observers with fixed coordinates $(x,y,z)$

after crossing the string horizon (in the frame of 'right' observers $(Y>0)$) :

- shift their positions (by rescaling coordinates):

$$\delta t = \lambda \zeta^t = -\lambda (p \cdot \bar{x}), \quad \delta x^i = \lambda \frac{\bar{x}^i}{a} (p \cdot \bar{x}),$$

$$(\bar{x}^i)^2 = a^2 e^{-2\chi}$$

$$(p \cdot \bar{x}) = \cos\alpha\,(x-a) + \sin\alpha\; y > 0, \ \text{if } Y < 0$$

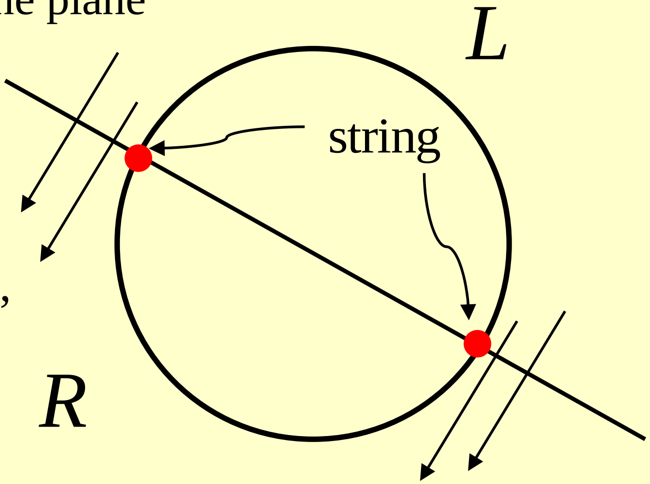
- acquire coordinate velocities in the direction orthogonal to the plane

of the string:

$$v^i = -\lambda p^i$$

as a result, some part of matter, after crossing the string horizon,

moves from $L$ region to $R$ region



# Kaiser-Stebbins effects (small energies)

'left' quanta change the energy when crossing the string horizon

$\delta\omega = \delta k_0 = \lambda L_\zeta k_0,$ where $k^\mu$ is 4-momentum of incoming $\gamma$

$$\delta\omega\Big|_{\text{string hor}} = \lambda\left[(p\cdot k) - \omega\frac{1}{a}(p\cdot\bar{x})\right]$$

temperature variation due to the string is

$$\boxed{\frac{\delta T}{T} = \lambda\, z_{\text{str}}(n)\left[(n\cdot p) - \frac{1}{a}(p\cdot\bar{x})\right] \quad , \quad k^i = \omega n^i}$$

$z_{\text{str}}$ – $z$-factor at a moment when 'left' photon crosses the string horizon

$$z_{\text{str}}(n,t) = \frac{1}{2}\left((2n_x - 1)e^{\chi} - 1\right)$$

# String image for de Sitter observer (5D picture)

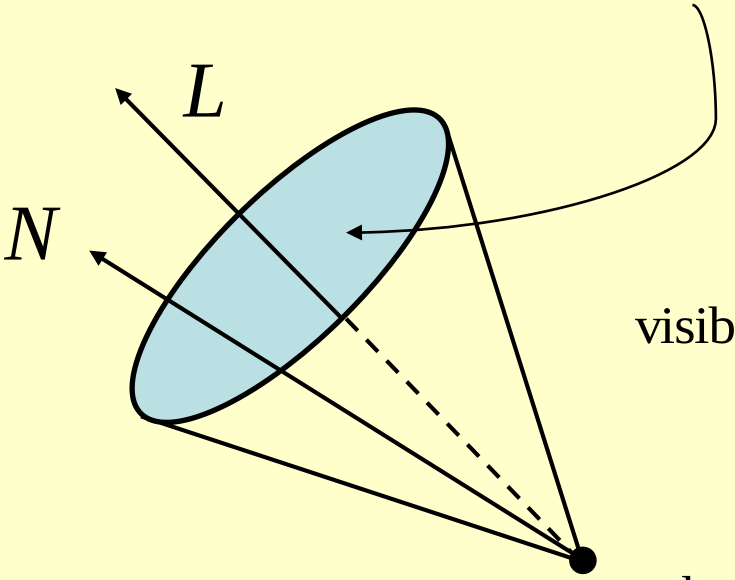
use embedding in $\mathbb{R}^{1,4}$ , trajectory of observer in $\mathbb{R}^{1,4}$ :

$$X^0(t) = a \sinh \chi, \quad X^p(t) = a \cosh \chi \, M^p, \text{ with } M^Y = \cos\alpha, M^2 = \sin\alpha,$$

$$\boxed{(N \cdot L) = \cos\Theta = \frac{Y_o}{\sqrt{Y_o^2 + T_o^2}}}$$ – visibility cone in $\mathbb{R}^{1,4}$

$$T_o = a \sinh \chi$$

$$Y_o = a \cos\alpha \cosh \chi$$



transformed quanta are inside

visibility cone

observer

# String image for de Sitter observer (4D picture)

use projection on de Sitter from $\mathbb{R}^{1,4}$ :

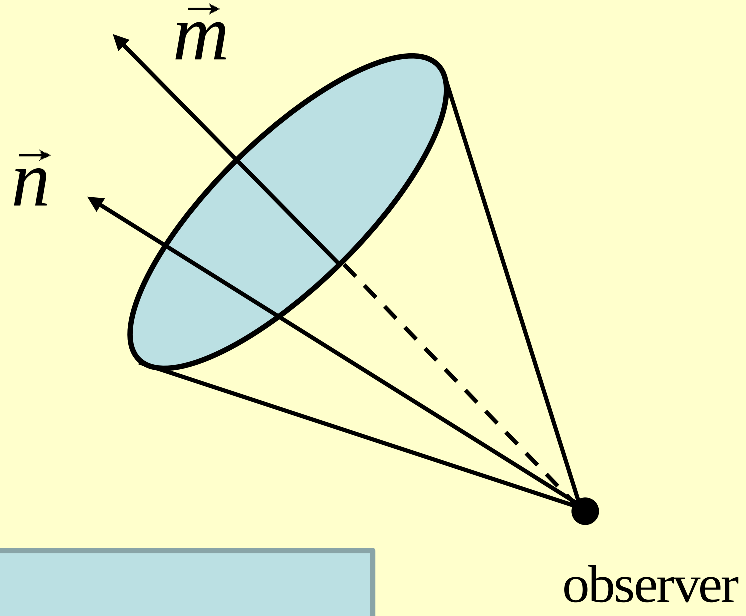
$$P_L^K = \delta_L^K - \frac{1}{a^2} X^K X_L + u^K u_L , \quad X^K , u^K - \text{are coordinates}$$

and velocity of observer

$$PN = \frac{n}{\cosh\chi} = N - \tanh\chi M ,$$

$$PL \to m = \frac{L - M\sin\Theta}{\sqrt{1 - \cos^2\alpha\sin\Theta}}$$

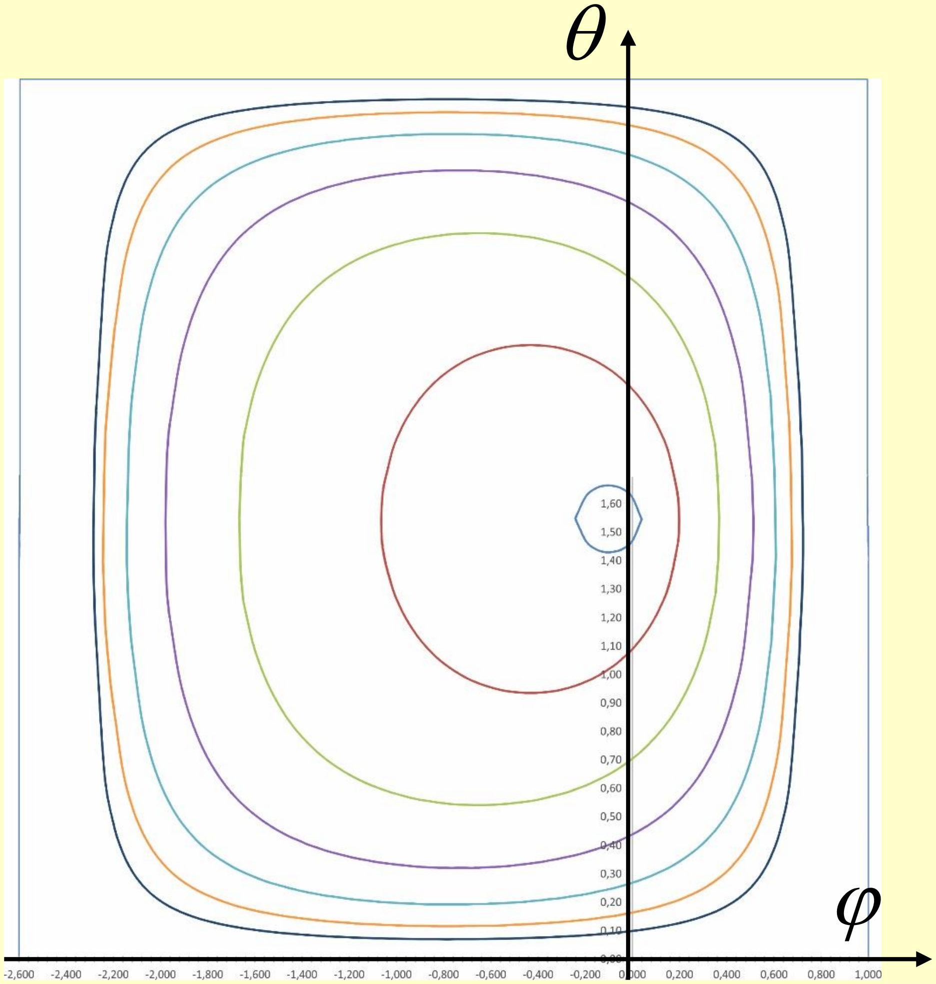
visibility cone in dS:



$$(\vec{n}\cdot\vec{m}) = \cos\hat{\theta} = \frac{\cos\alpha}{\sqrt{\cos^2\alpha\cosh^2\chi + \sin^2\alpha\sinh^2\chi}}$$

# Image of a string (continued)

time evolution of the "image" (for an observer at rest)



time evolution of the "image"

for an observer at $r = 0$ for $\alpha = \pi/4$

and $\chi = 0,1;\ 0,5;\ 1;\ 1,5;\ 2;\ 2,5;\ 3$

NB: only part of the image is observed (cutoff due to finite time of recombination)

# Summary

- massless strings (MCS) in a flat space: the beauty of the special relativity;

- the technique, wake and Kaiser-Stebbins effects for MCS;

- field theory;

- MCS in expanding universe (focus on de Sitter);

- effects, observations

thank you for attention