

Special Lecture on Gravitational Waves

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Dmitri Fursaev
Dubna U & JINR

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February 11, 2016: an official announcement about the direct detection of gravitational waves by

Laser Interferometer Gravitational-Wave Observatory

event GW150914 (dated September 14, 2015)

the cause of the event is a merger of two black holes, 29 and 36 solar masses, at a distance of 1,3 billions of light years from the Earth, final black hole is 62 times larger than the Sun

maximal luminosity was 50 times larger than the radiation of the visible part of the Universe

Prehistory

- 1916 - A. Einstein predicts gravity waves (GW) in GR;
- 1969 - first attempts to detect GW by Joseph Weber (USA) and by Vladimir Braginsky (USSR) by using bar detectors (resonant antenna);
- 1962 and 1972 an optical method is proposed for detecting GW;
- 1974 - Hulse and Taylor discover a binary pulsar, whose period changes in accord with the prediction by GW radiation theory in GR;
- 1994 - LIGO \$272 million construction starts ;
- 2002 – 2010 – LIGO first search for GW (no GW signals was found);
- 2002 - GEO600 starts operate in Germany, 2007 – VIRGO in Italy;
- 2010 –2015 \$205 million LIGO upgrade;
- 2015 – Advanced LIGO 1st observation run.

2 more events:

“Boxing Day” event GW151226 (dated December 26, 2015) from a black hole binary with 14 and 8 solar masses, 1.4 billions l.y. away, final black hole 21 times larger than the Sun

event GW170104 (dated January 04, 2017), from a black hole binary with 19 and 32 solar masses, 3 billions l.y. away, final black hole 49 times larger than the Sun

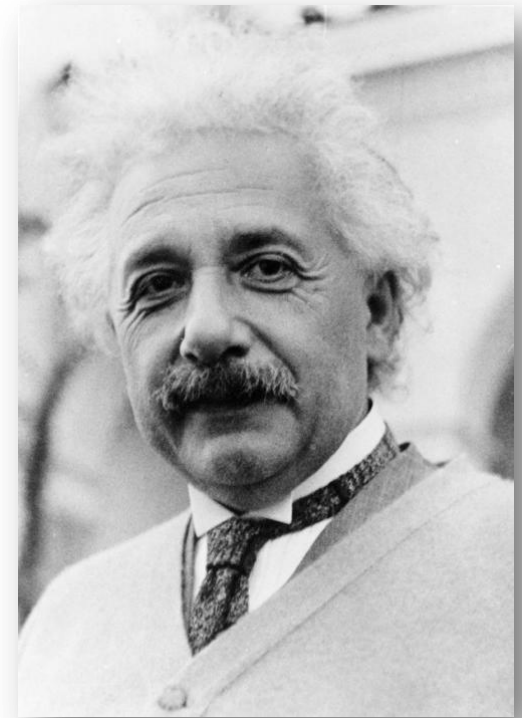
Topics of the talk:

- What are the gravitational waves (GW)?
- GW detectors. History...
- LIGO
- BH data from event GW150914
- Using GW for fundamental discoveries?

Gravitational waves and the Einstein Relativity

Einstein's General Relativity (1915)

- Einstein's theory is determined by a set of postulates (the Einstein principle of equivalence);
- space and time cannot be considered separately, and are united in a continuum (a manifold);
- gravitational fields can be described in terms of metric and geometry of a space-time;
- masses and localized energies are sources for gravitational field (and curvature) ;
- curved stationary space-times may also exist without matter (since the theory is nonlinear);
- moving massive bodies may result in ripples of geometry = gravitational waves



A. Einstein

Experimental tests of GR

(GR cannot compete with the Standard Model!)

- classical tests of GR (geodesics, weak gravitation, little information about gravity equations);
- “Gravity Probe B” experiments: confirmation of some subtle predictions related to geometry around rotating bodies;
- double pulsars;
- applications to cosmology: mounting evidence that GR may explain the Early Universe (FRW, inflation);
- **No indications that theory fails**, BUT precision in cosmology is on the level of few percents;
- Dark Matter and Dark Energy may have an explanation outside the Einstein gravity: many alternative gravity theories are on the market;

Why detection of GW is important?

- it is a proof of one of the fundamental predictions of the theory;
- tests of GR in case of strong gravity (GW are ripples over the geometry which appear from 'storms' - massive compact sources!);
- astrophysical events which cannot be seen by other methods;
- possible restrictions on models of modified gravity;
- GW astronomy in future – a new channel of the information about the Universe;
- new technologies (classical and quantum measurements, optics and lasers; geology and geodesy, cryogenic technologies, numerical methods in GR);

**Gravitational Waves – “ripples” of
space-time geometry**

Geometry of space-time?

The key object in the Riemannian geometry is a metric:

Metric is a set of rules which defines a scalar product for two vectors at a point, it is used to determine the curvature.

Metric has 2 times more components than a vector (it is 4×4 matrix);

Gravitational waves are solutions to the Einstein equations which look as wave type perturbations of the metric (and curvature), 'ripples of the space-time geometry'

Metric and interval

$g_{\mu\nu}(x)$ ← metric tensor

$V, U \rightarrow (V \cdot U)_g = V^\mu g_{\mu\nu} U^\nu$ ← scalar product

an interval between two events at points x^μ и $x^\mu + dx^\mu$

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$ ← no curvature

The metric allows one to determine time intervals

$d\tau$ - an interval of time as measured by an observer

$$d\tau^2 = -c^{-2} ds^2 \Big|_{\text{on trajectory}} = -g_{00} dt^2$$

$$-g_{00}(R) = 1 - \frac{2MG}{c^2 R} \leftarrow \text{at a distance } R \text{ from a body}$$

$$\frac{d\tau_1}{d\tau_2} = \frac{g_{00}(R_1)}{g_{00}(R_2)} \simeq 1 - \frac{2MG}{c^2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

gravity 'slows' the time (decreases frequencies)

the Pound and Rebka experiment (1960)

One measures physical distances with the metric

$L = \sqrt{dl^2}$ - physical distance between two nearby observers

$$dl^2 = h_{ij}(x)dx^i dx^j$$

$h_{ij}(x)$ - 3-dimensional metric tensor

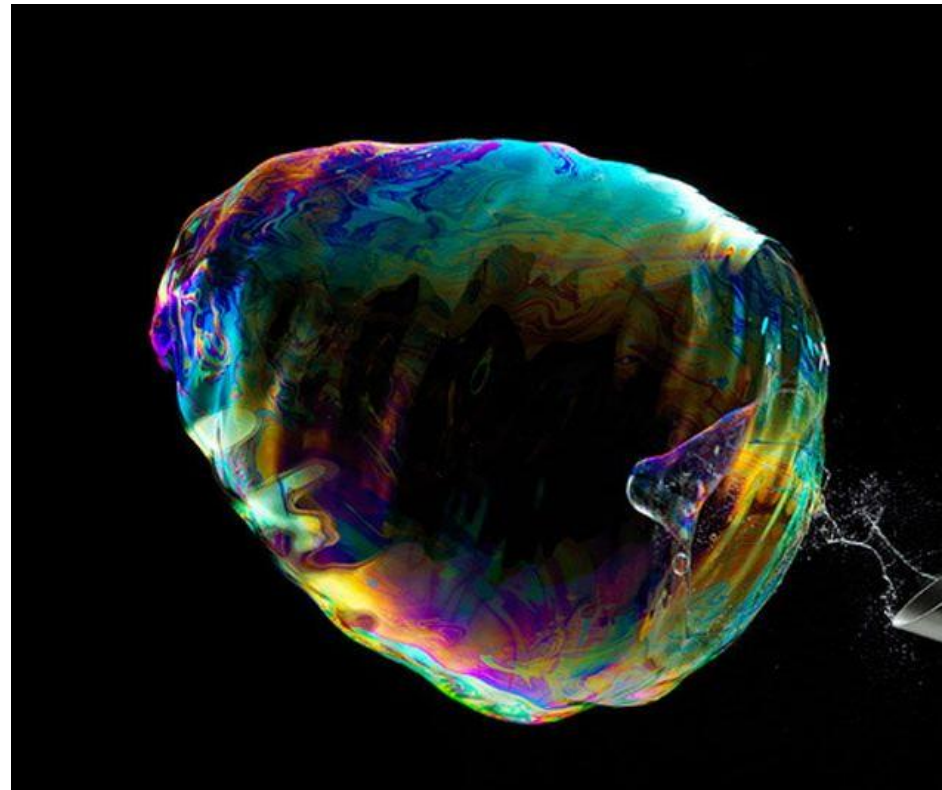
$$h_{ij} = g_{ij} - \frac{g_{oi}g_{oj}}{g_{00}}$$

length of a rotating disc with radius R , and angular velocity Ω

$$L = 2\pi R / \sqrt{1 - \frac{R^2\Omega^2}{c^2}} > 2\pi R$$

gravitational field and inertial forces change 3D geometry

Gravity waves – wave type perturbations of the metric



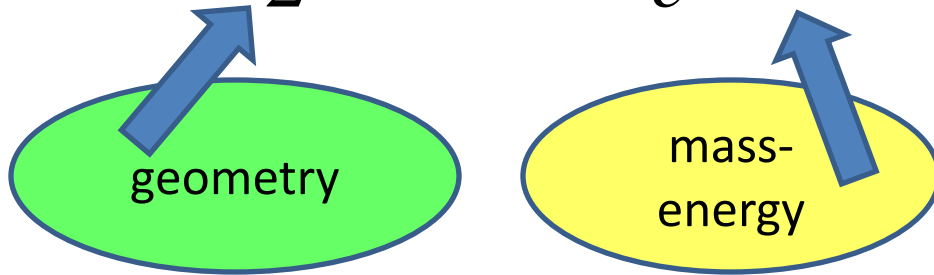
$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$$

$\bar{g}_{\mu\nu}(x)$ - a background (slowly changing) metric

$h_{\mu\nu}(x)$ - a perturbation, which propagates as a wave

Einstein Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} \quad \leftarrow \text{stress-energy tensor}$$

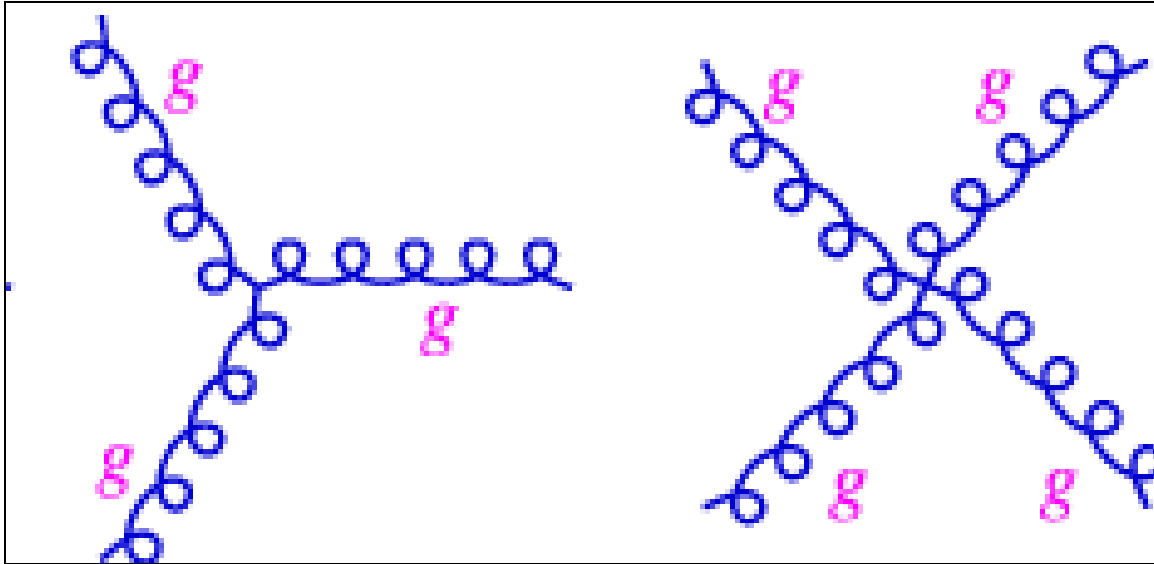


$$R = \frac{1}{2} g^{\mu\nu} g^{\lambda\rho} \partial_\nu \partial_\rho g_{\mu\lambda} + \dots \quad - \text{curvature}$$

$g^{\mu\nu}$ - inverse matrices of $g_{\mu\nu}$

- Einstein equations contain second derivatives of the metric
- Einstein equations are non-linear

Gravity equations are *essentially* non-linear
(as opposed to the Standard Model eqs)



+ «vertices» with any numbers of legs

if quantized, the theory is non-renormalisable

«self-interactions» of gravitons allow pure vacuum solutions to the Einstein eqs., like Schwarzschild black holes (analogous of instanton solutions in the Yang-Mills theory, but in the real space-time)

GW are solutions to linearised Einstein eqs.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

metric is almost flat

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \leftarrow \text{linearized theory}$$

$\eta_{\mu\nu}$ - Minkowsky metric (Speacial Relativity)

$$\left(c^{-2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right) h_{\mu\nu}(ct - x) \simeq \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

linearized equations for a GW, moving along x axis

GW change distances between bodies

let L_0 be a distance between 2 points on the z axis;

if a GW moves along x orthogonally to z , the distance varies as:

$$L(t) = \sqrt{dl^2} = \sqrt{(1 + h_{zz}(x - ct))\Delta z^2} \simeq \left(1 + \frac{1}{2}h_{zz}(x - ct)\right)L_0,$$

where h_{zz} is a component of the metric in the z direction

(amplitude of GW)

strain - relative change of interferometer's arm length

$$\frac{\Delta L}{L} \simeq \frac{1}{2}h_{zz} \text{ - is determined by the amplitude of GW}$$

How small is the strain?

for GW150914 the strain is just $\frac{\delta L}{L} \sim 10^{-21}$,

for LIGO arm $L = 4 \text{ km}$ the absolute change is 4×10^{-18} meters,
which is 200 times less the size of the proton!

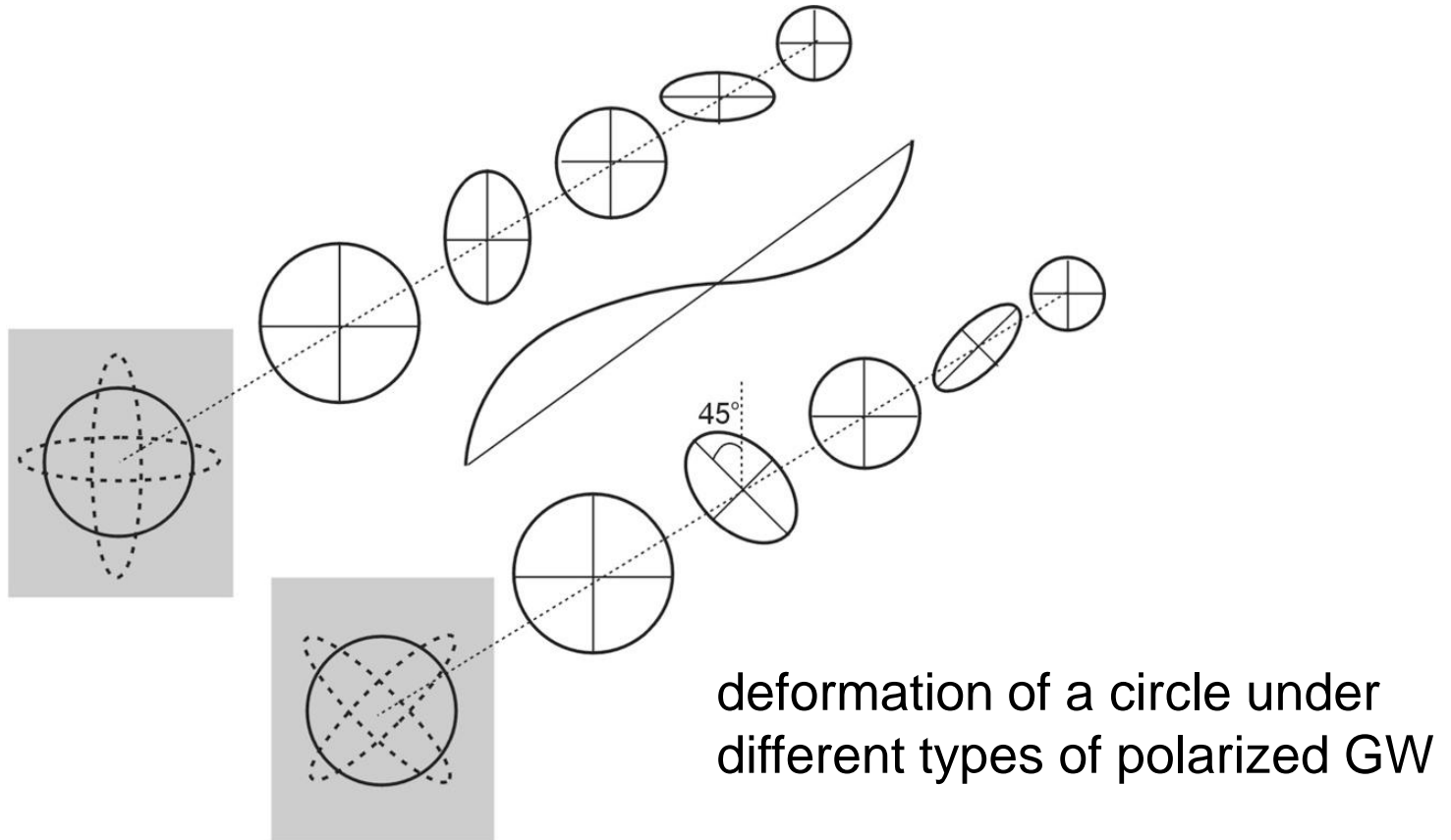
The challenge is to construct detectors sensitive enough to recognize such small variations in the noise. The strategy is to improve detector's sensitivity and compare signals in detectors with known templates (based on numerical simulations in GR).

features of GW

- GW move with the velocity of light (a consequence of covariance of GR);
- GW take away energy and angular momentum from a source;
- wave amplitude decreases as inverse distance to the source (a solution to the second order PDE, same as EM) ;
- standard frequency shifts (Doppler, gravitational,...);
- GW interaction with matter is weak (difficult to detect, but carried information is not destroyed);
- sources of GW are compact super massive bodies (binary systems such as BH-BH, BH-NS, NS-NS);
- there is a hypothesis that primordial GW might appear in the inflationary Universe (can be detected from CMB) - still waits a proof

Other features of GW

- GW have spin 2 (EM waves have spin 1)
- GW can be polarized, 2 types of linear polarization, circular polarization is possible



Quadrupole nature of GW radiation

$$L_{dipole} = \frac{2}{3} \ddot{d}_e^2 \quad \leftarrow \text{luminosity of electric dipole}$$

$$\vec{d}_e = \sum_{\text{charges}} e_k \vec{x}_k \quad \leftarrow \text{electric dipole moment (DM)}$$

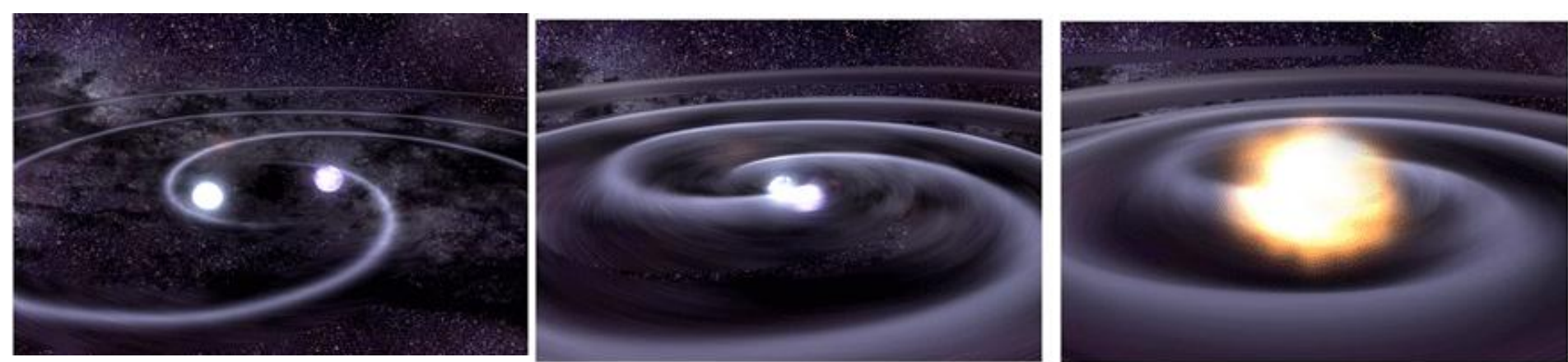
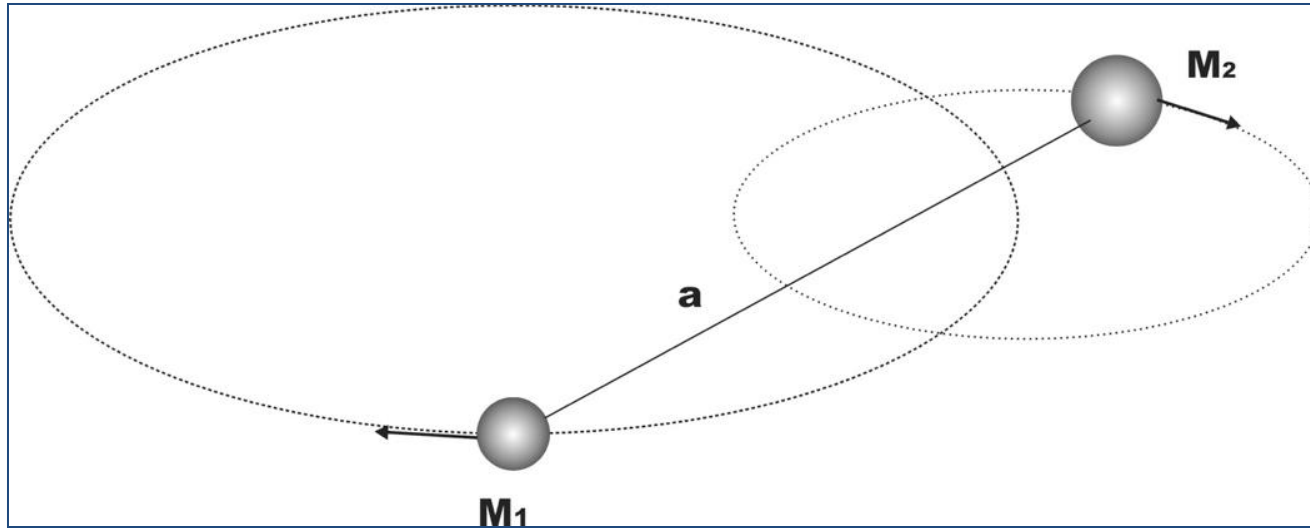
$$\vec{d}_{gr} = \sum_{\text{masses}} m_k \vec{x}_k \quad \leftarrow \text{gravitational analog of DM}$$

$$\frac{d}{dt} \vec{d}_{gr} = \sum_{\text{masses}} \vec{p}_k \quad \leftarrow \text{total momentum, hence } \frac{d^2}{dt^2} \vec{d}_{gr} = 0$$

$$L_{quadrupole} = \frac{G}{5c^5} \ddot{Q}_{kj} \ddot{Q}_{kj} \quad \leftarrow \text{luminosity of gravirtational quadrupole}$$

$$Q_{kj} = \sum_{\text{masses}} m_n \left(x_{nk} x_{nj} - \frac{1}{3} \delta_{kj} r^2 \right) \quad \leftarrow \text{quadrupole moment (tensor)}$$

Binary systems as typical sources of GW radiation



peak of the energy emission is on the stage of merging

Some basic estimates for binaries

Let Ω be orbital angular frequency,

$a(t) \sim \cos \Omega t$ – distance from a star (or BH) in a binary to a centre,

contribution to the quadrupole moment $Q \sim a^2(t) \sim \cos^2 \Omega t$

$$h(t) \sim \ddot{Q} \sim \cos 2\Omega t$$

each orbit makes 2 gravitational wave cycles!

$$\text{GW frequency: } f_{GW} = 2\Omega / (2\pi)$$

One can use 3d Kepler's law: $\Omega^2 \sim \frac{MG_N}{a^3}$

and relate f_{GW} to the mass M and size a of the binary;

for a binary $M = 10M_{\odot}$, $a = 100 \text{ km}$, $f_{GW} \sim 1 \text{ kHz}$

Why the strain is so small?

from the linearized Einstein equations one gets (far from the source):

$$h_{\mu\nu}(\vec{x}, t) = -4G_N \int_V d^3x' \frac{T_{\mu\nu}(ct - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} = \frac{2G_N}{Rc^4} \ddot{Q}_{\mu\nu}(ct - r)$$

R – distance from the source to an observer

$$h \sim \frac{G_N}{Rc^4} \ddot{Q} \sim \frac{G_N}{Rc^4} Ma^2 \Omega^2 \sim \frac{G_N}{Rc^4} Ma^2 \frac{G_N M}{a^3}$$

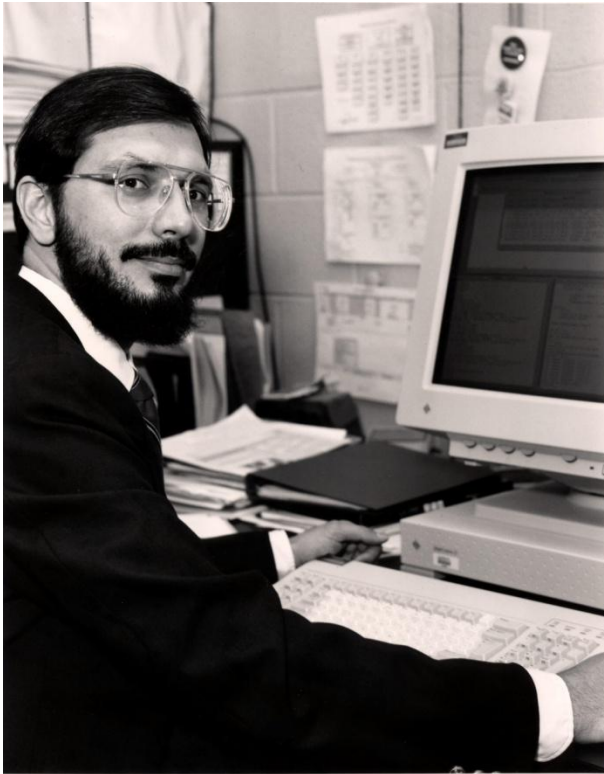
$$h \sim \frac{r_H^2}{aR}, \quad r_H = \frac{2G_N M}{c^2} \text{ – gravitational radius of the binary}$$

$$r_H \approx 2.95 (M / M_\odot) km, \quad 1 \text{ light year} \approx 1.6 \cdot 10^{11} km,$$

for a binary $M = 10M_\odot$, $a = 100 km$, $R = 10^9 l.y.$

$$h \sim 10^{-20}$$

**the Hulse-Taylor binary pulsar
(discovered in 1974)**



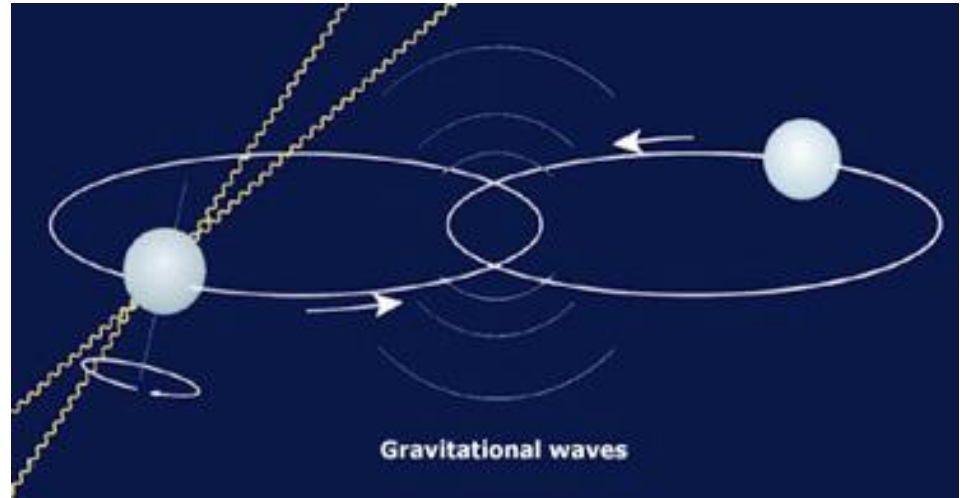
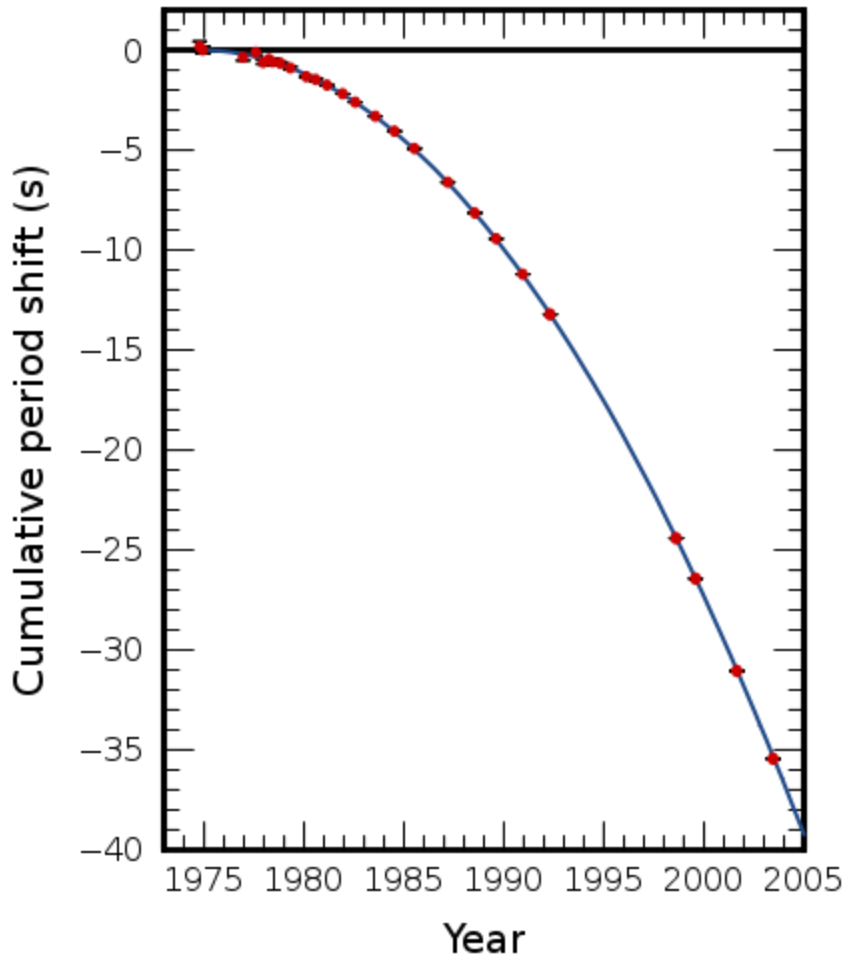
R.A. Hulse (USA)



J.H. Taylor (USA)

First astrophysical evidence for GW

Nobel Prize in 1993



binary system of 2 neutron stars:

- distance to the Earth - 10 thousand light years;

- masses of stars are about 1.4 Solar mass each,

- rotation period 7 hours 45 min,

decreased due to GW by 40 seconds

over 30 years;

-frequency of emitted GW 7×10^{-5} Hz,

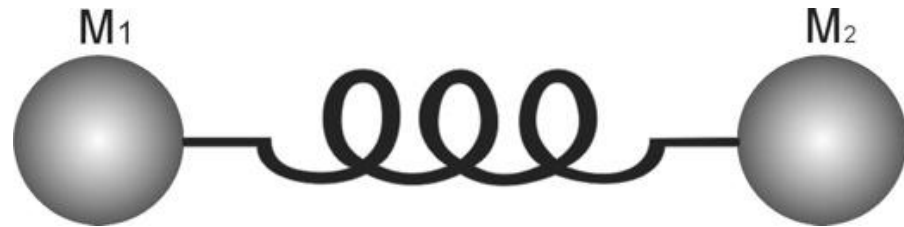
amplitude $h \sim 10^{-23}$

GW detectors

J. Weber and first attempts to detect GW (1970's)

Bar detectors (Weber bars) can detect GW with frequencies close to their resonance frequencies

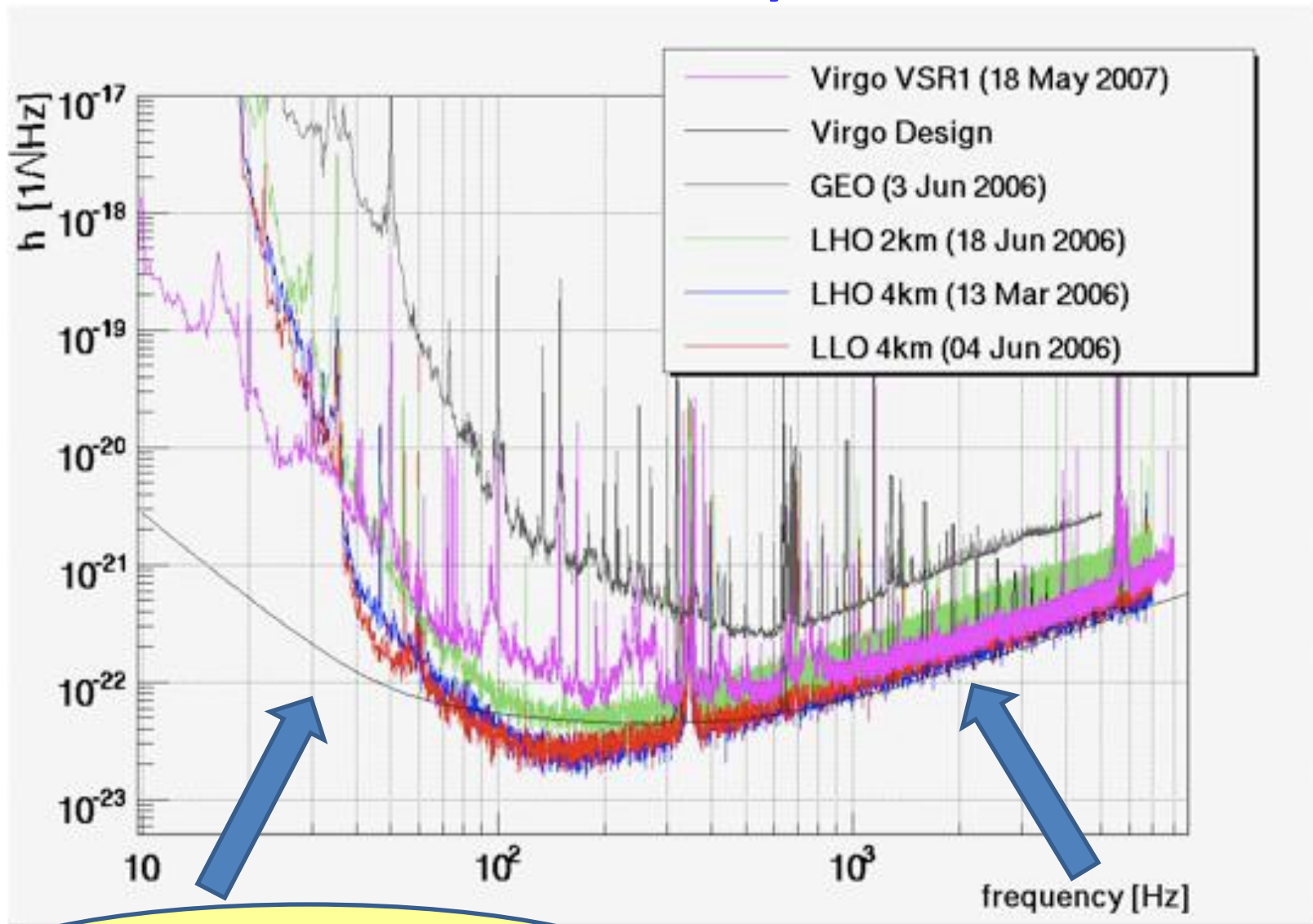
A device consists of 2×1 meters aluminum cylinders, sensitive to change of $10^{-16} m$



One of the problems: narrow band width (800 - 1000 Hz)



Sensitivity



Seismic noise

photon shot noise
(fluctuating number
of photons in a laser)

Gravitational Observatory LIGO

Two interferometers are located at Hanford, Washington, and Livingston, Louisiana, USA



Distance between Hanford and Livingston is about 3000 km
(which takes 10 milliseconds for a light travel)

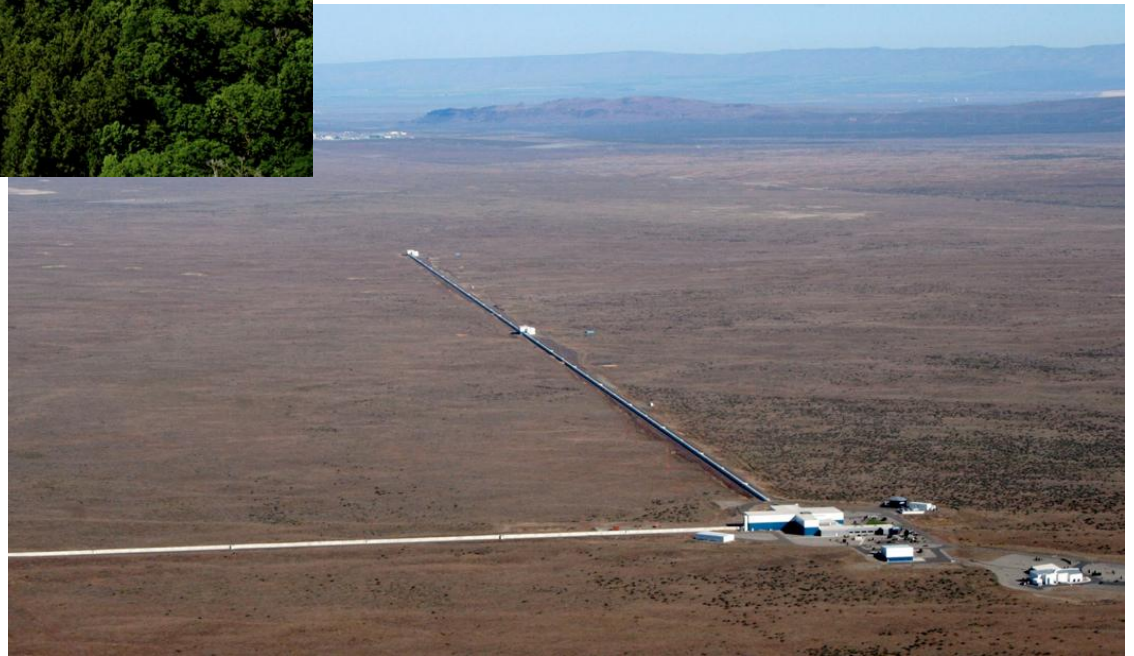
Basic facts about LIGO

- LIGO is operated by Caltech and MIT, about 40 people work at each observatory site;
- world wide cooperation of about 80 research centers from different countries;
- vacuum chambers are second in the world, by size (after LHC);
- test masses are suspended sapphire mirrors 30 kg each, distance between masses is 4 km;
- sensitivity of detectors increased 10 times after 5 year upgrade of the machines;
- as a result, an observable volume of the Universe accessible for LIGO increased 1000 times);

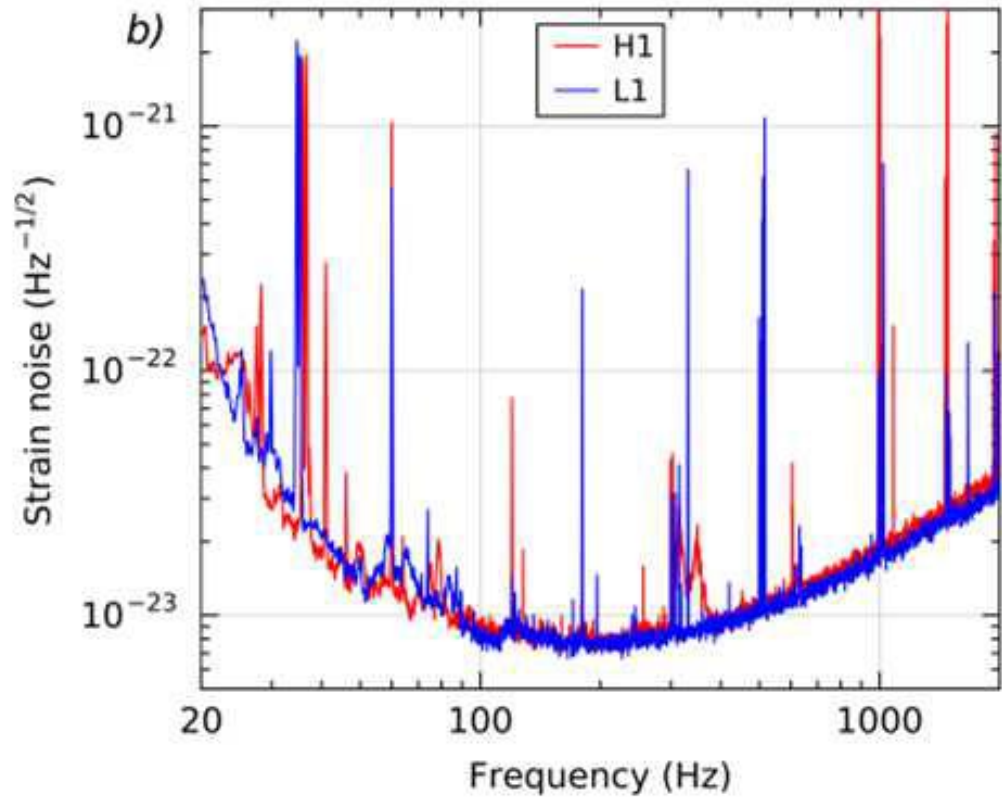
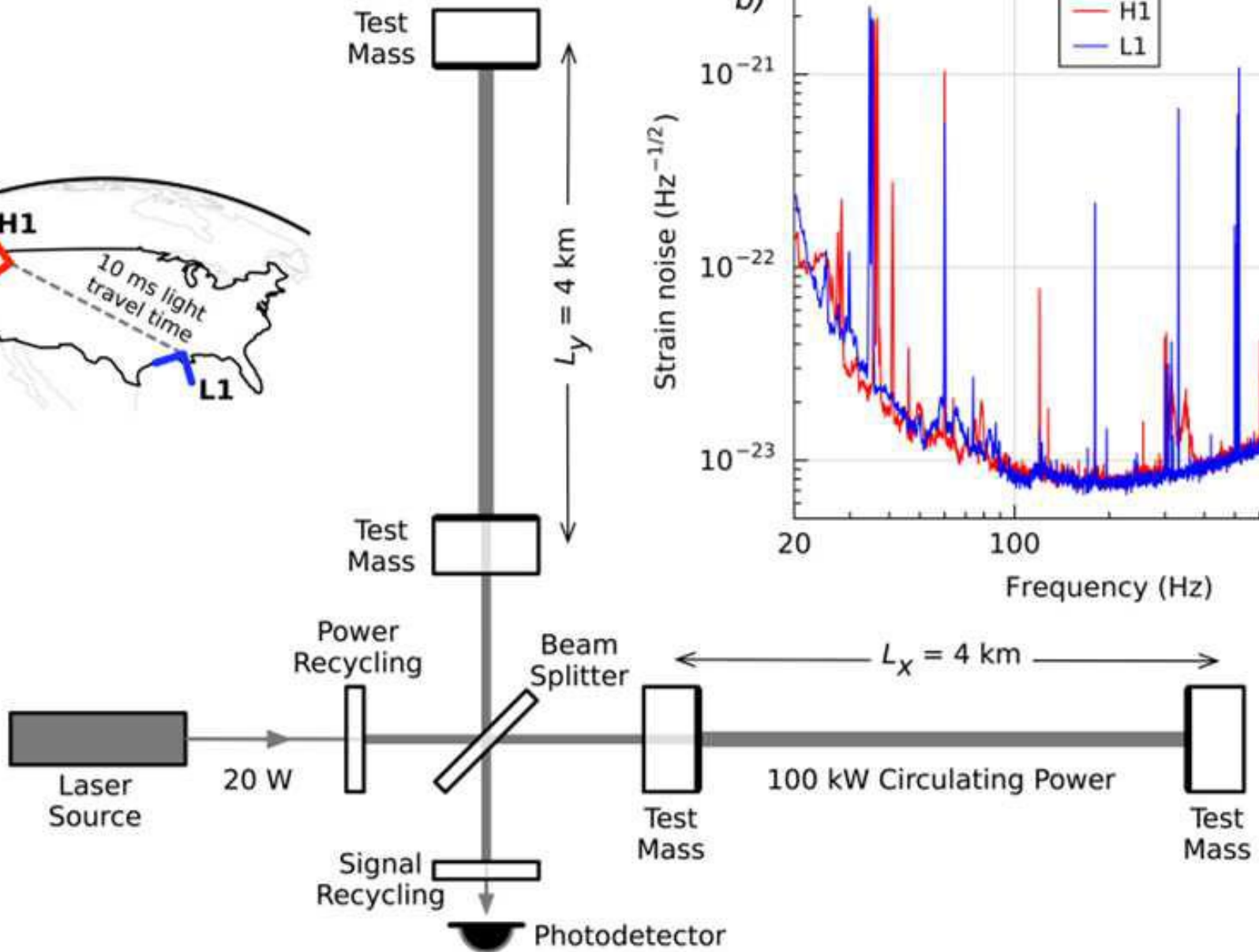
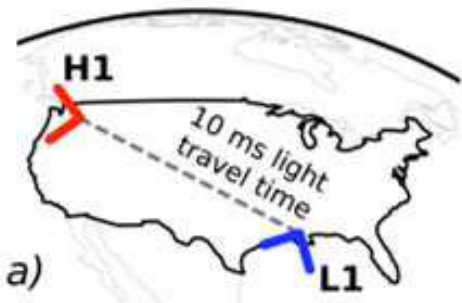
LIGO



the length of arms - 4 km



LIGO detector's sensitivity



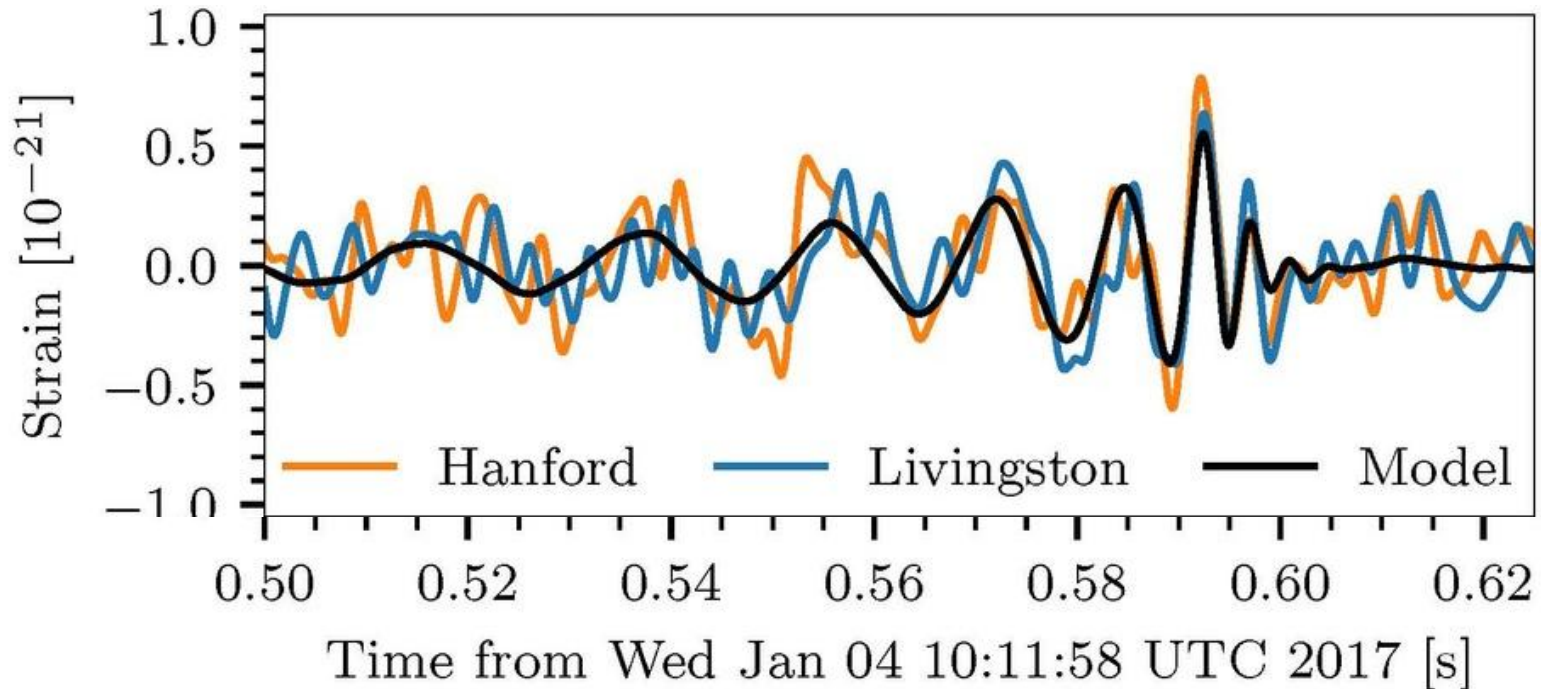
GW150914

The event happened on 14 September 2015 during a preliminary engineering run, four days before 1st observing run planned on 18 September 2015;

0.007- second delay between signals at Hanford and Livingston;

Injections systems producing “blind injections” (false signals) were not operating.

Signals were visible with a naked eye



visible part of the signal lasts about 0.15 s;
signal to noise ratio - 24;

it would take 200 000 years for the noise to mimic such or similar signals

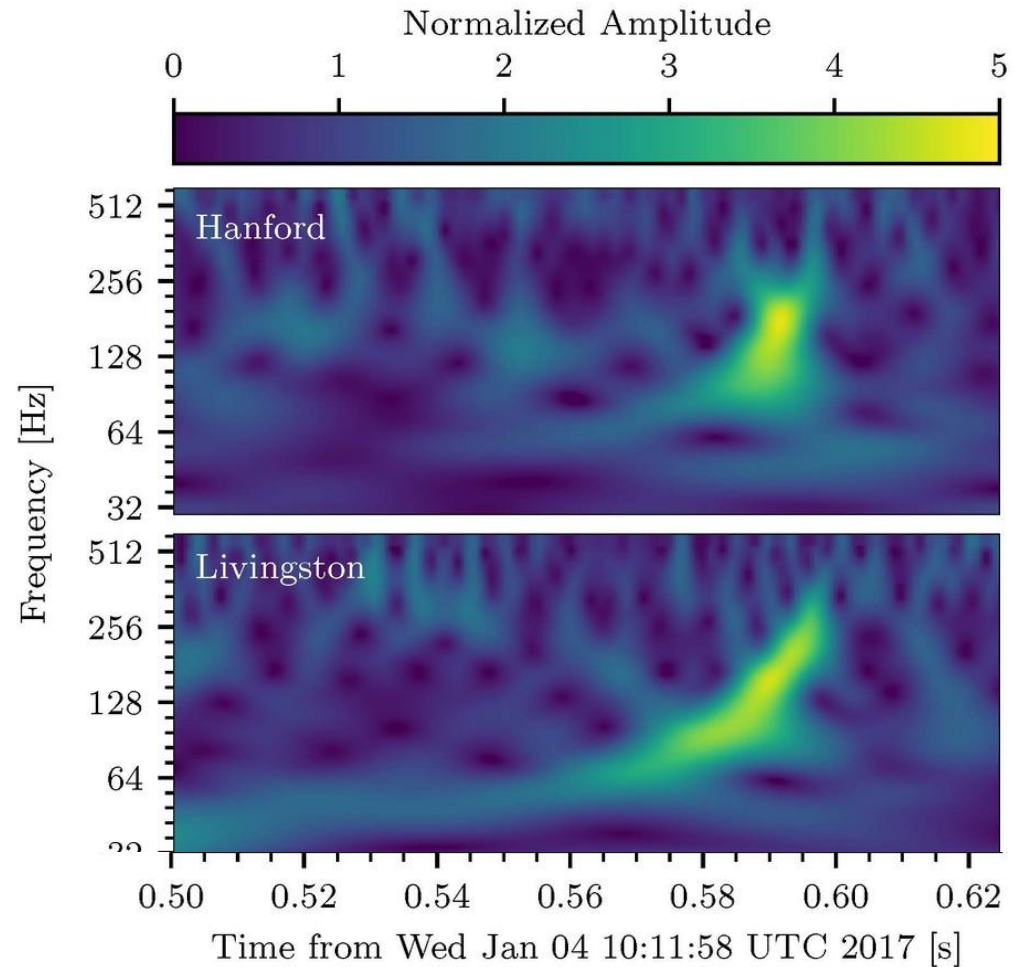
The increase of signal frequency in time (“chirp”)

frequency change band from
35 to 150 Hz;

signals are real and from GW:

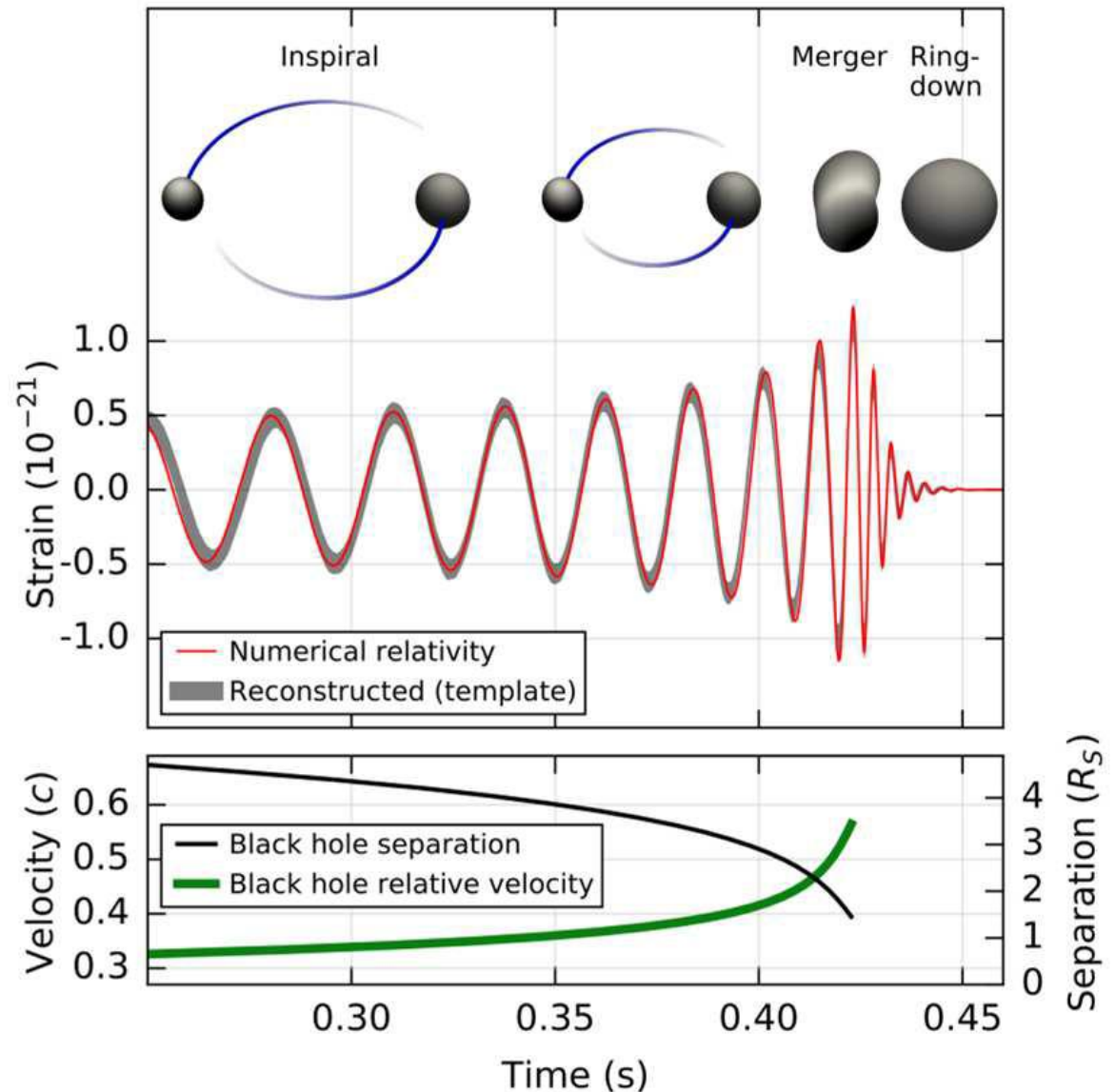
-they are unmodeled signals,
not the noise (stat. signifi-
cance 4.6 sigma);

- fit numerical simulation of the
signal from a merging (5.1
sigma) – matched filtering
search;



Interpretation: GW from merging BH in a binary

sharp drop of the signal at the end corresponds to a horizon formation



Extracting binary parameters from signals

– $f_{GW} = 150 \text{ Hz}$ GW frequency at a peak,

– $\Omega = 75 \text{ Hz}$ orbital frequency,

– \dot{f}_{GW} rate of frequency increase,

– $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \approx 30 M_{\odot}$ - chirp mass of the binary

$$\mathcal{M} \sim \frac{c^3}{G_N} \left((f_{GW})^{-11} (\dot{f}_{GW})^3 \right)^{1/5}$$

– $m_1, m_2 \approx 35 M_{\odot}$ - masses of binary components;

– $a \approx 350 \text{ km}$ - separation of components (from Kepler's law)

– $E \approx \frac{G_N m_i^2}{2a} \approx 3 M_{\odot}$ - radiated energy

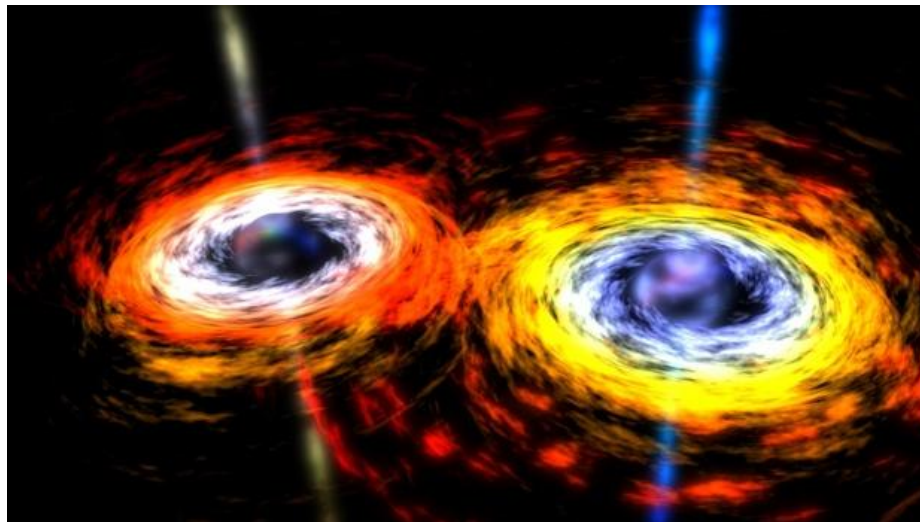
it is a black hole - black hole binary!

objects with masses $35M_{\odot}$ have gravitational radius about 100 km, separation 350 km

- ordinary stars 10^6 km in size and not heavy enough;
- white dwarfs are 10^4 km, not heavy;
- neutron stars are not heavy enough;

black holes are the only candidates!

waveform of the signal is consistent with numerical relativity predictions



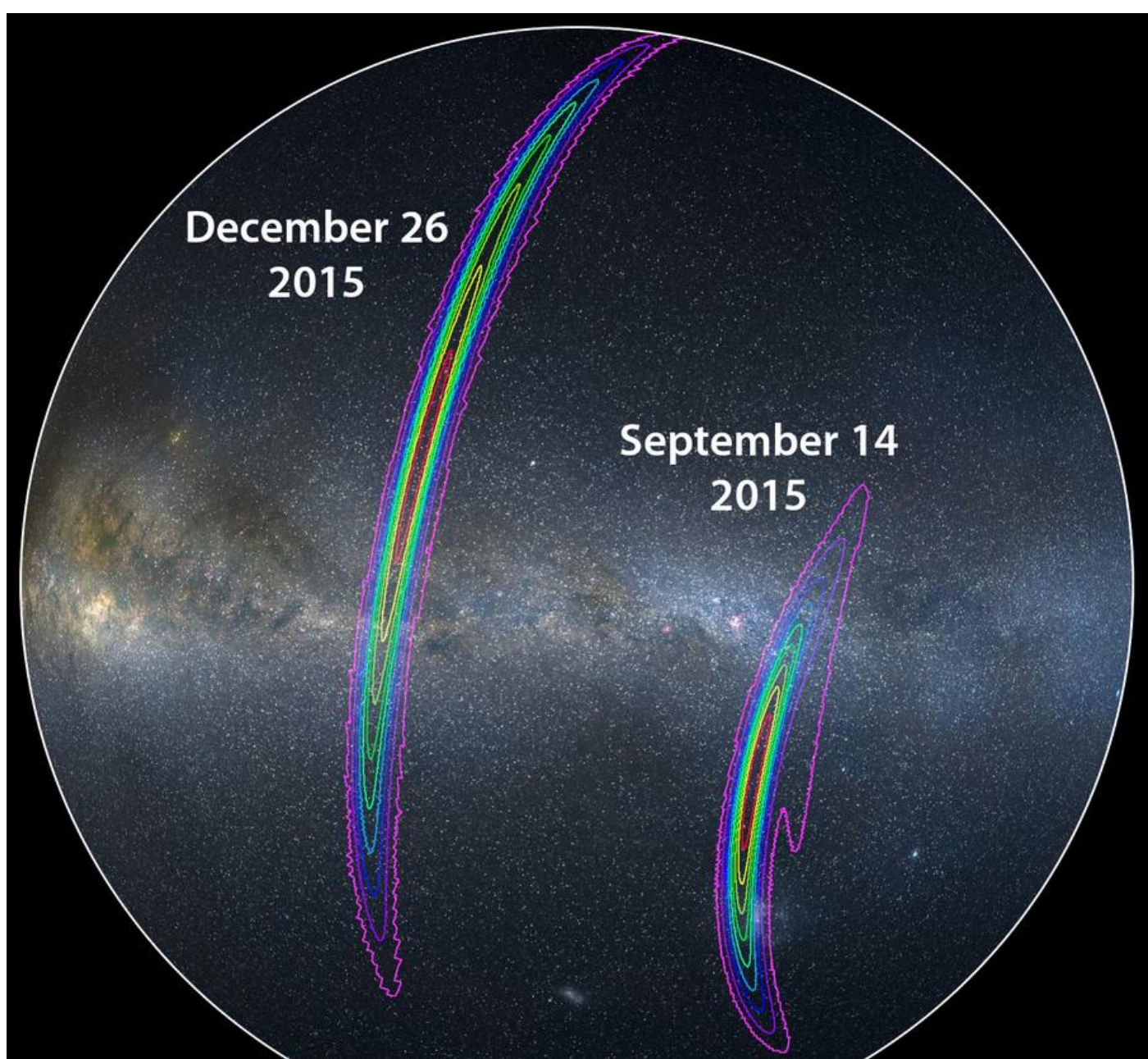
- merging of 2 black holes, 29 and 36 Solar masses, initially 350 km apart,
- distance to the Earth is 1.3 billions light years ($z \sim 0.1$),
- maximal luminosity is 50 times larger than luminosity of the visible part of the Universe,
- velocities of black holes at the moment of merging are 0,5 speed of light;
- mass of the final black hole is 62 Solar masses.

The event is perfect to study effects of strong gravity!

important consequences

- evidence for black hole binaries (not seen by other methods);
- GW150914 is in agreement with GR;
- the binaries can merge for the time less than the life time of the Universe;
- evidence for black holes with masses larger than 25 Solar masses (intermediate class of black holes);
- such events may happen 2 - 400 times a year in a part of the Universe with the syze 1 Gpc ;
- upper limit on the mass of the graviton $m_g < 1,2 \times 10^{-22} \text{ eV}$,

Future projects



2 detectors cannot locate the source, more detectors are needed

nearest and distant future

- global network of gravitational detectors:
 - advanced VIRGO (Italy), 3km arm;
 - KAGRA (Japan)
 - 3d LIGO (India)
- space based detectors in a distant future (eLISA): GW from super massive BH;
- LIGO 03 run: sensitivity factor 3 better,
- a golden age of GW astronomy is coming: up to 100 events per year
- learn more about spin and mass of binaries, locate their positions, combine with observations by means of “traditional” astronomy
- snapshot of the Universe at the age of a fraction of a second by detecting primordial GW (an analog of CMB which yields a snapshot at the age of 380 000 years)

Thank you for attention!